THE JOURNAL OF FINANCE • VOL. LX, NO. 4 • AUGUST 2005

Strategic Behavior and Underpricing in Uniform Price Auctions: Evidence from Finnish Treasury Auctions

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ABSTRACT

We contribute to the debate on the optimal design of multiunit auctions by developing and testing robust implications of the leading theory of uniform price auctions on the bid distributions submitted by individual bidders. The theory, which emphasizes market power, has little support in a data set of Finnish Treasury auctions. A reason may be that the Treasury acts strategically by determining supply after observing bids, apparently treating the auctions as a repeated game between itself and primary dealers. Bidder behavior and underpricing react to the volatility of bond returns in a way that suggests bidders adjust for the winner's curse.

ECONOMISTS AND POLICYMAKERS HAVE DEBATED the optimal design of multiunit auctions for decades. Much of the debate has been shaped by Friedman's (1960) proposition that the U.S. Treasury could decrease funding costs by using uniform price rather than discriminatory price auctions. In both auction formats, individual bidders submit collections of bids (demand schedules) and the securities are awarded in the order of descending price until supply is exhausted. In uniform auctions, winning bidders pay the market-clearing (or, stop-out) price

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for all units awarded; in discriminatory auctions winning bidders pay what they bid. In this paper, we contribute to the debate by examining empirically the leading theory of uniform auctions. We also shed light on the economic factors that influence bidding and auction performance. Our results suggest that the manner in which a seller implements the auction can have significant impact on performance.

The theoretical auctions literature has advanced arguments both for and against uniform auctions. In support of Friedman's view, analogous to the result by Milgrom and Weber (1982) on second-price versus first-price auctions, it has been argued that uniform auctions reduce the winner's curse relative to discriminatory auctions and that they therefore generate more revenue (see, e.g., Milgrom (1989) and Bikhchandani and Huang (1993)). However, models by Wilson (1979) and Back and Zender (1993), which explicitly incorporate bidders' demand functions, reach the opposite conclusion. When bidders submit downward-sloping demand schedules, each bidder faces an upward-sloping residual supply curve over which he is a monopsonist. Under uniform pricing, this market power is shown to be optimally exercised by submitting a decreasing demand function so that the auctioned securities will be underpriced relative to the secondary market. In a sense, bidders manipulate the clearing price by submitting a few low bids. Equilibrium underpricing can be arbitrarily large. The Wilson/Back and Zender model is cast in terms of risk-neutral players, but the same market power effect is also at the heart of a model by Kyle (1989) with risk-averse players (see also Wang and Zender (2002)). It has been argued that since uniform auctions, but not discriminatory auctions, are susceptible to underpricing from monopsonistic market power, more revenue can be raised by using discriminatory auctions (see particularly Back and Zender (1993)).

The empirical literature has compared the revenue-raising abilities of uniform and discriminatory auctions by looking at the level of underpricing relative to benchmarks such as contemporaneous when-issued yields or secondary market prices. There is growing evidence that underpricing is smaller in uniform treasury auctions than in discriminatory treasury auctions (Umlauf (1993), Nyborg and Sundaresan (1996), Malvey and Archibald (1998), Goldreich (2003)). Thus it seems that the theoretically predicted market power equilibria fail to materialize in practice. However, objections may be raised to this conclusion. First, any level of underpricing is consistent with the theory; there are numerous equilibria. One interpretation of the evidence is that bidders simply coordinate on equilibria with a relatively low underpricing, on average. Second, underpricing as measured by empiricists is not necessarily an accurate reflection of revenue; for example, the benchmark may reflect the (expected) auction outcome (Nyborg and Sundaresan (1996)). In this paper, we will therefore examine the market power theory not by looking at underpricing, but rather by examining whether observed bidder behavior is consistent with the theory.

We employ a bidder-level data set of uniform treasury auctions from Finland over the period 1992 to 1999. These auctions would appear to be particularly vulnerable to market power because of the small number of bidders (between 5 and 10). In addition, the variation in the number of bidders combined with the fact that the number is known prior to bidding makes these auctions an ideal laboratory for testing the market power theory.

To test the theory, we develop a new methodology, which focuses on how market power may be exercised by individual bidders. The basic idea is to compare the theoretical demand schedules with those that bidders actually submit. We view demand schedules as distributions and compute summary statistics of the theoretical and empirical bid distributions at the individual bidder level. The validity of the theory is assessed by comparing the predicted and observed statistics, particularly by checking whether the empirical statistics react to exogenous variables as predicted by the theory. While the tests are derived in the context of uniform auctions, the methodology can be employed in other contexts in which a theory delivers predictions on the specifications of demand or supply functions.

We show that a central property of the market power theory is that at the individual bidder level, the theoretical bid distribution exhibits negative skewness. Intuitively, this is because bidders have incentives to submit small, low bids in order to reduce the price they pay for all the units they win. The skewness is also predicted to decrease with the number of bidders. We find the opposite. While the empirical bid distributions tend to be negatively skewed when there are few bidders (5 to 8), the average skewness becomes significantly positive when there are many bidders (9 to 10). Also very troublesome for the market power theory is the fact that we cannot reject the null hypothesis that bid shading (the discount) and underpricing are unaffected by the number of bidders. Consistent with bidders exercising some market power, albeit less than suggested by the theory, we find that demand per bidder is increasing in the number of bidders.

The rejection of the market power theory leaves us with two important questions. First, what is the primary driver behind bidder behavior and auction performance? Second, what can explain the rejection?

In our sample, the variable that has the most significant economic impact on bidder behavior and underpricing across auctions is volatility. An increase in volatility leads to larger discounts and more underpricing, reduced demand, and increased dispersion. These findings parallel those of Nyborg, Rydqvist, and Sundaresan (2002) on discriminatory Swedish Treasury auctions (see also Cammack (1991)). It would therefore appear that the same basic economic forces are at work in uniform treasury auctions in Finland as in discriminatory treasury auctions in Sweden. One possibility is that bidders have private information and rationally adjust for the winner's curse, as discussed by Nyborg et al. (2002).[1] This view is consistent with the finding cited above that uniform auctions typically have less underpricing than discriminatory auctions.

Strategic behavior by the seller may explain why the market power theory is rejected. A special feature of the Finnish auctions is that the Treasury determines supply after observing the bids. We document that the Finnish Treasury has never chosen supply to maximize revenue given the bids in an auction. The Treasury even cancelled a few auctions because bids were not deemed to be

[1] Ausubel (1997) also discusses the winner's curse in multiunit auctions.

sufficiently high. This behavior suggests that the seller thinks of the auction as a repeated game in which the bids in subsequent auctions can be influenced by rejecting revenue increasing bids in the current auction. Bidders may respond by submitting bids that are more aggressive than what is predicted by theoretical models with a nonstrategic seller.

The rest of the paper is organized as follows. The Finnish Treasury market and the data are described in Section I. The market power theory of uniform auctions is surveyed in Section II with an emphasis on drawing out testable restrictions. Section III examines bidder behavior empirically and tests the theory. Section IV analyzes the seller's strategic behavior. Section V compares bidding and underpricing in uniform and discriminatory treasury auctions. Section VI concludes.

I. Institutional Background and Data

A. The Finnish Treasury Bond Market

The Finnish Treasury started issuing securities in 1991. This was motivated by the need to finance the government budget deficit. The top left panel in Figure 1 shows that the deficit was very large during the recession in the early 1990s, when GDP growth was negative, but turned into a surplus toward the end of the decade. As a result, the Finnish Treasury stopped issuing new securities in 1999 and has been buying back securities since 2000. The bottom left panel shows the annual number of treasury bond auctions (the dark columns) and the number of occasions the Treasury offers additional securities for sale by fixed-price tender (the light columns). The frequency of auctions is approximately evenly distributed over time except in the beginning and the end of the period. The total number of auctions is 232 and the number of fixed-price tenders is 48.[2] The top right panel shows the annual average auction size, which is increasing over time at an average rate of 24% per year. The large auction amounts in the late 1990s, when the net budget balance was positive, were used to refinance maturing debt and to exchange foreign with domestic debt. The bottom right panel shows that the number of primary dealers varies over time. The primary dealers have an exclusive right and an obligation to bid in the auctions. Finland introduced a primary dealer system in August 1992. The 25 auctions prior to this date were open to anybody.[3]

Regular auctions are held every second Thursday, when one or two treasury bonds are sold simultaneously. The 232 auctions are spread out over 204 calendar days; 176 days with a single security for sale, and 28 days with two securities. Thirteen auctions are first issues of a new security and 219 are reopenings of existing securities that already trade in the secondary market. The

[2] The fixed-price tenders are held the day after the auction. In these, winning bidders in the auction get the right to purchase additional securities up to 30% of their auction awards. The price is the auction's stop-out price or higher. The Treasury also sells T-bills with up to 1 year to maturity.

[3] For a more detailed description of the Finnish Treasury bond market, see Keloharju et al. (2002).

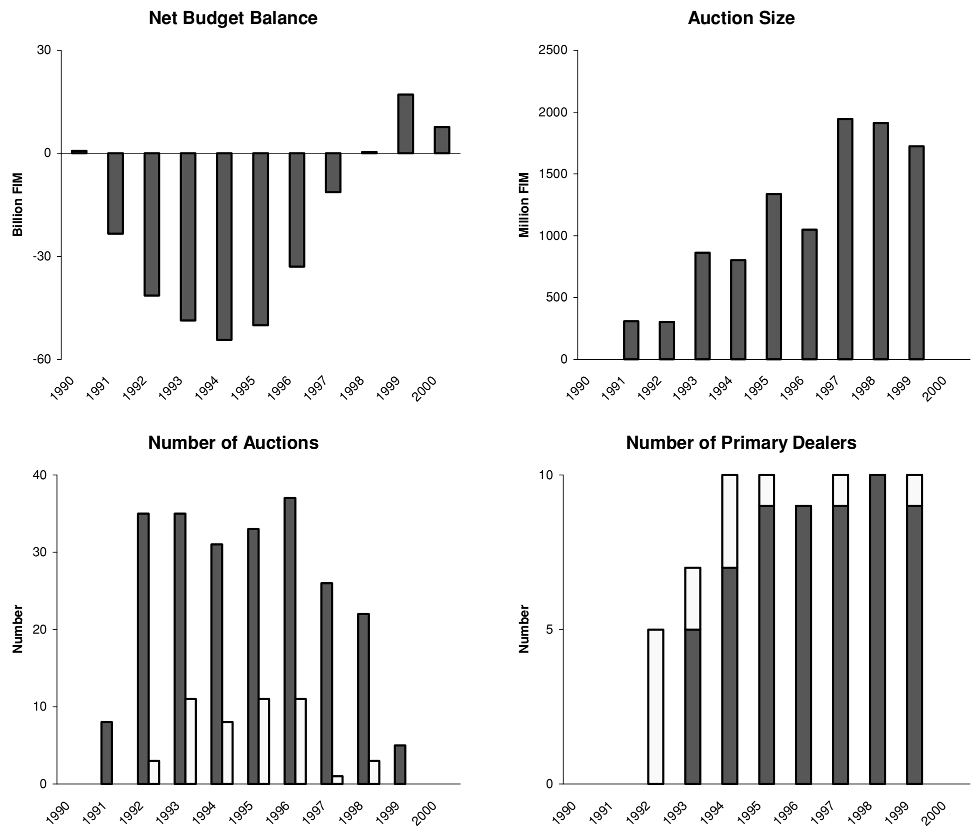

Figure 1. Finnish government net budget balance and treasury bond auctions 1990 to 2000. Top left: Annual net balance of the Finnish government budget. Bottom left: Annual number of treasury bond auctions (dark bars) along with the number of fixed-price tender (white bars). Top right: Annual average auction size. An approximate exchange rate is one dollar for 6 markkas. Bottom right: Number of primary dealers from August 1992 to 1999. Within each year, the minimum number is represented by the dark portion and the maximum by the dark and the light portions together.

bonds are noncallable, have annual coupons, and have tenors between 2 and 15 years.

The auction format is sealed, multiple bid, and uniform price. Awarded bidders pay the stop-out price, which is the price of the lowest awarded bid. Bids are submitted by phone before 1:00 p.m. and confirmed by fax afterward. Auction awards are announced at 1:30 p.m. Individual bids are expressed in price per 100 markka of face value, with a tick size of 0.05 markka before May 7, 1998 and 0.02 markka thereafter. The quantity multiple is one million markka of face value.

One week before the auction, the Treasury announces which securities will be offered for sale, but not the amount. Supply is determined after observing the bids. From 1998, the Treasury announces the maximum amount. The Finnish Treasury does not have an explicit policy regarding the choice of quantity and stop-out price, and they do not operate with preannounced reservation prices. Conversations with one Treasury official revealed that their actual choices are influenced by (1) the long-term revenue target, (2) market conditions, (3) the Treasury's own opinion about the true market price, and (4) an unwillingness to spoil the market by accepting very low bids.

The secondary market for a new security opens immediately after the first auction. When trading becomes sufficiently active, a committee consisting of the Treasury and the primary dealers promotes the security to benchmark status. The dealers must report all their transactions in benchmark bonds to the Bank of Finland, and they must also post bid and ask quotes. Usually, the dealers start posting quotes some time before the benchmark designation. The bond loses its benchmark status 1 year before maturity.

B. Bid Distribution Data

For this study, the Finnish Treasury has produced a data set, which contains all the bids in 231 of the 232 auctions. The last auction is missing. Each record provides the price per 100 markka of face value, the yield to maturity, the face value demanded at that price, and a two-digit dealer code. The code is constant throughout the sample. We shall focus on the 206 auctions under the primary dealer system when the number of bidders is fixed prior to each auction. In these auctions, the total number of individual bidder demand schedules is 1,702 and the total number of bids is 4,583.[4] The average number of bids per demand schedule is 2.7, the standard deviation 1.7, the median 2, and the mode 1. The maximum is 14 bids in one demand schedule.

C. Secondary Market Data

The primary dealers post bid and ask yield quotes on the Bloomberg screen for all benchmark securities. The posted bid and ask quotes are binding for 10 million markkas. The primary dealers, in consultation with the Finnish Treasury, determine the maximum posted spread. At any point in time, this is a constant across different bonds. During the sample period, the maximum posted spread has been revised only five times and varies between 2, 3, 5, and 10 basis points (bp). Customers may get better quotes in private negotiations with primary dealers, but such quotes are not posted.

The Bank of Finland collects the posted bid quotes at 1:00 p.m. every day and computes the primary dealer average. Combining the bid time series with the maximum posted spread series, we can construct a time series of posted quotes for each bond. This time series covers 181 of the 206 auctions. The missing data

[4] One outlying bid is excluded from our data set. It is the lower in a demand schedule of two bids, and submitted at a price that is more than 35 quantity-weighted standard deviations below the quantity-weighted mean of the other bids in the auction.

are from the first few auctions of each security before dealers start posting quotes. The Bank of Finland also collects daily transaction yields for trades between dealers and customers, categorized by whether the dealer buys or sells. For each category, the Bank of Finland computes the equally weighted average yield and aggregate trading volume. The average buy and sell yields fall within the posted quotes. The time series of transactions data covers 153 of the 206 auctions.

In the empirical analysis, we want to compare the bids and the stop-out price of an auction with the secondary market transaction price $P_j(Y_j)$, where Y_j denotes the transaction yield of security j at the time of the auction. We use the Bank of Finland bond yield series described above to approximate Y_j and then compute $P_j(Y_j)$. Our approach is to adjust the 1:00 p.m. average posted bid yield quotes by the systematic deviation between posted quotes and transactions yields. We first pool the time-series and cross-section data and employ the 7,058 daily observations from August 1992 to April 1999 for which we have complete bid and ask quotes as well as average dealer–customer buy and sell transaction yields. We compute two spreads, the bid quote minus the buy yield and the sell yield minus the ask quote. Since the average buy yield minus the bid quote (1.01 bp) is less than the average sell yield minus the ask quote (1.82 bp), we conclude that transaction yields are biased toward the bid quote. The bias suggests that a reasonable approximation of Y_j is the bid quote minus an adjustment for the general level of transactions yields relative to the bid quote itself. We therefore compute a third spread, namely, the bid quote minus the transaction sell yield, which we refer to as the dealer's markup, since it reflects a markup of the price that dealers get from customers relative to the dealer's bid quote. The idea behind computing the markup using the average dealer–customer sell yield is that dealers buy in the auction to sell in the secondary market.

As a normalization, we condition the dealer's markup and the two other spreads on the maximum posted spread. Table I shows that our constructed spreads increase with the size of the posted spread, which falls over time from

Table I

Maximum Posted Spread and Constructed Spreads

Average yield spreads in basis points. The maximum posted spread is from an agreement between the primary dealers and the Treasury. The buy and sell yields are the average daily transaction yields for purchases from and sales to customers. The dealer's markup is the posted bid quote less the sell yield. Daily data from August 1992 to April 1999.

	Maximum Posted Spread (bp)			
	2	3	5	10
Bid minus buy	0.41	0.84	1.38	1.85
Sell minus ask	1.06	1.26	1.86	6.16
Dealer's markup	0.94	1.74	3.14	3.84
No. Obs	1,186	3,283	1,969	620

10 to 2 bp. We estimate Y_j by subtracting the conditional dealer's markup from the bid quote.[5] For example, if the posted bid quote at 1:00 p.m. on the auction day is 5% and the posted spread 2 bp, we infer Y_j to be $5 - 0.0094 = 4.9906\%$. While this means that we are measuring Y_j and thus P_j with error, we believe the error is reduced relative to relying on the bid quote or the midpoint of the spread. When bid quotes are missing, the observation is dropped from our data set. We do not attempt to extrapolate the missing secondary market yields from the sparse term structure data in Finland.

II. Theory of Bidder Behavior in Uniform Auctions

In this section, we review the market power theory of uniform auctions. The emphasis is on drawing out testable empirical implications. We consider in turn the cases that bidders are risk neutral and that they are risk averse.

A. Market Power When Bidders Are Risk Neutral

The case of risk-neutral bidders was first explored by Wilson (1979) and later by Back and Zender (1993) who introduce supply uncertainty. In their model, there are N identical bidders, each of whom can buy the entire auction. The auction size, Q, may be random and is at most $Q_{\max}$. Bidders have identical valuations of $\bar{v}$ per unit. One can think of $\bar{v}$ as the expected secondary market price.[6] Wilson and Back and Zender show that there are numerous equilibria in which bidders submit decreasing demand functions, which result in underpricing, that is, a stop-out price below $\bar{v}$.

Equilibrium underpricing arises from the price–quantity tradeoff faced by each bidder when all the other bidders submit decreasing demand functions. In this case, a bidder can increase his share of the auction by submitting a higher demand function, but this comes at the expense of raising the stop-out price and thereby decreasing the profit per unit he buys. For a given stop-out price, the quantity a bidder receives is the residual supply—the quantity left over after other bidders' demand has been filled.[7] So each bidder is essentially maximizing his profit against an increasing residual supply curve. In short, when the bidders submit downward-sloping demand functions, each of them

[5] We have not attempted other more complicated procedures to estimate the markup, for example, using lags, volatility, or number of dealers to forecast it. One reason is that the autocorrelation in the dealer's markup time series is only 0.08, so there is little to gain from using lags. Another reason is that the transactions data is missing for about 11% of all trading days. This would give rise to other estimation problems.

[6] Our exposition of the Wilson/Back and Zender model assumes that bidders do not have private information about the secondary market price. Back and Zender develop their basic argument in a private information framework, but in equilibrium bidders do not use it. So market power underpricing equilibria may exist also when bidders are privately informed. Wilson provides an example with private information in which the stop-out price is perfectly revealing, but underpricing still occurs because of market power.

[7] In the underpricing equilibria, demand functions are strictly decreasing so rationing is not an issue. The importance of this is discussed by Kremer and Nyborg (2004a,b).

is a monopsonist with respect to the residual supply curve he faces. The underpricing equilibria are cemented by the fact that each bidder can optimally exercise his monopsonistic market power by submitting a decreasing demand function. So the underpricing equilibria are characterized by a sort of complicit agreement among bidders to give each other monopsonistic market power and thus create underpricing.

When the auction size is known, the first-order condition of a bidder's price–quantity tradeoff needs to be satisfied only at the stop-out price itself. As a result, there are numerous underpricing equilibria. When supply is uncertain and exogenous, however, the first-order condition must be satisfied along the set of all possible stop-out prices. As a result, there is a unique class of supply-uncertainty robust demand functions, as found by Back and Zender (see also Kremer and Nyborg (2004a)). We shall focus on these equilibria since bidders in the Finnish auctions do not know the supply when they submit their bids. The unique supply-uncertainty robust equilibria are given by

$$q(p) = a\left(1 - \frac{p}{\bar{v}}\right)^{\frac{1}{N-1}}, \tag{1}$$

where $a \geq Q_{\max}/N$ is the quantity demanded at a price of zero. Given a, the inverse demand curve is

$$p(q) = \left[1 - \left(\frac{q}{a}\right)^{N-1}\right]\bar{v}. \tag{2}$$

Under (1), demand at a price of zero is a, while demand is zero at prices of $\bar{v}$ and higher. For $N \geq 3$, the demand schedule exhibits strict concavity. The intuition is related to the price–quantity tradeoff faced by bidders: given that the stop-out price is below $\bar{v}$, each bidder would appear to have an incentive to bid more aggressively to get a bigger share of the auction. So it must be that a large increase in quantity can only be achieved by a large increase in price. Furthermore, it must be that a small decrease in price will result in a large decrease in quantity; otherwise, bidders would have an incentive to be more passive. This is essentially a convexity condition on the residual supply and therefore a concavity condition on individual demand functions, especially since this must be satisfied along the continuum of possible stop-out prices (see Kremer and Nyborg (2004a) for further discussion).

Under (1), the stop-out price, which equates demand and supply, is

$$p_0 = \left[1 - \left(\frac{Q}{aN}\right)^{N-1}\right]\bar{v}, \tag{3}$$

where Q is the realized auction size. Total revenue from the auction is thus p_0Q and depends upon $\bar{v}$, a, N, and Q. Depending on a, underpricing can fall anywhere between 0 and $\bar{v} - r$, where $r \geq 0$ is the reservation price.

B. Market Power When Bidders Are Risk Averse

B.1. CARA Utility and Linear Equilibria

Kyle (1989) presents a model in which bidders have CARA utility with risk aversion coefficient ρ. The post-auction value of the auctioned security, $\tilde{v}$, is normally distributed with expectation $\bar{v}$ and variance σ^2. We shall focus on the special case of his model where players do not have private information and supply is positive. Kyle's model then becomes one in which risk-averse bidders choose demand schedules as strategies in the same way that risk-neutral bidders do in the Wilson/Back and Zender model. By stripping away private information, we thus emphasize the implications of monopsonistic market power and risk bearing. Kyle demonstrates that there is a unique linear equilibrium, which is robust to supply uncertainty, namely,

$$q(p) = \frac{N-2}{N-1}\frac{\bar{v}-p}{\rho\sigma^2}. \tag{4}$$

We provide a straightforward derivation of this equilibrium in Appendix A. The inverse demand schedule is

$$p(q) = \bar{v} - \frac{N-1}{N-2}\rho\sigma^2 q. \tag{5}$$

To isolate the effect of market power from the effect of risk aversion, we can compare (4) to the corresponding Marshallian (or nonstrategic) demand schedule under CARA utility. Standard arguments show that the Marshallian schedule is the linear function

$$q(p) = \frac{\bar{v}-p}{\rho\sigma^2}, \tag{6}$$

with inverse

$$p(q) = \bar{v} - \rho\sigma^2 q. \tag{7}$$

The negative slope is a result of risk aversion, and linearity is a result of CARA utility and normality. The strategic inverse demand schedule (5) is located below the Marshallian inverse (7). As N goes to infinity, the strategic equilibrium converges to the competitive one. As in the case of risk-neutral bidders, this illustrates that a feature of supply-uncertainty robust equilibria is that market power diminishes when N increases and eventually vanishes in the limit.

Under the strategic demand schedule, (4), the stop-out price is

$$p_0 = \bar{v} - \frac{N-1}{N-2}\frac{\rho\sigma^2 Q}{N}. \tag{8}$$

Under the nonstrategic schedule, (6), we get the competitive price

$$p_0 = \bar{v} - \frac{\rho\sigma^2 Q}{N}. \tag{9}$$

These formulas show that underpricing, $\bar{v} - p_0$, is larger when bidders exercise market power. Furthermore, underpricing increases with the risk aversion coefficient and the amount of aggregate risk, $\sigma^2 Q$, that must be borne by a given number of bidders. An increase in N reduces underpricing primarily because more bidders share the aggregate risk, but also because market power is reduced.

B.2. CARA Utility and Nonlinear Equilibria

A surprising result is that Kyle's equilibrium does not converge to that of Back and Zender as the risk aversion coefficient goes to zero. The reason for this can be understood by looking at the general solution to Kyle's model, which has been shown by Wang and Zender (2002) to be (in inverse form)[8]

$$p(q) = \left[1 - \left(\frac{q}{a}\right)^{N-1}\right]\bar{v} - \left[1 - \left(\frac{q}{a}\right)^{N-2}\right]\left(\frac{N-1}{N-2}\right)\rho\sigma^2 q, \qquad (10)$$

where a is an arbitrary positive constant. These equilibria have the intuitive property that as ρ goes to 0, they converge to Back and Zender's equilibria (2). Thus the first term in (10) is a pure reflection of market power. The second term can be interpreted as a discount related to risk bearing. The parameter a plays an important role. As long as $a \leq \frac{\bar{v}}{\rho\sigma^2}$, equation (10) is strictly decreasing and $p(a) = 0$; that is, demand at a price of zero equals a, as in Back and Zender's equilibrium. If $a = \frac{N-2}{N-1}\frac{\bar{v}}{\rho\sigma^2}$, equation (10) reduces to Kyle's linear equilibrium.

C. Graphical Illustration of Market Power Equilibria

The four pictures in Figure 2 illustrate some of the key features and comparative statics of the different market power equilibria. In particular, they show how the location and shape of the different equilibrium demand functions are affected by changes in the number of bidders (top two pictures) and volatility (bottom two pictures).

The top two pictures in Figure 2 show that as the number of bidders increases from 5 to 10, the equilibrium demand function shifts out toward the Marshallian one.[9] Intuitively, as more bidders enter the auction, competition reduces the scope to exercise market power. In Kyle's equilibrium (top right), where bidders are risk averse, total quantity demanded per bidder increases with N. In Back and Zender's equilibrium (top left), where bidders are risk neutral, we see that concavity becomes more pronounced when N increases. This concavity effect is also a feature of Wang and Zender's equilibrium (not shown).

Volatility matters when bidders are risk averse. The bottom two pictures in Figure 2 illustrate the intuitive result that as volatility falls, risk-averse

[8] See Appendix A for a derivation of (10).

[9] In the Back and Zender figure, where bidders are risk neutral, the Marshallian inverse demand function is simply a horizontal line at $p = \bar{v} = 1$.

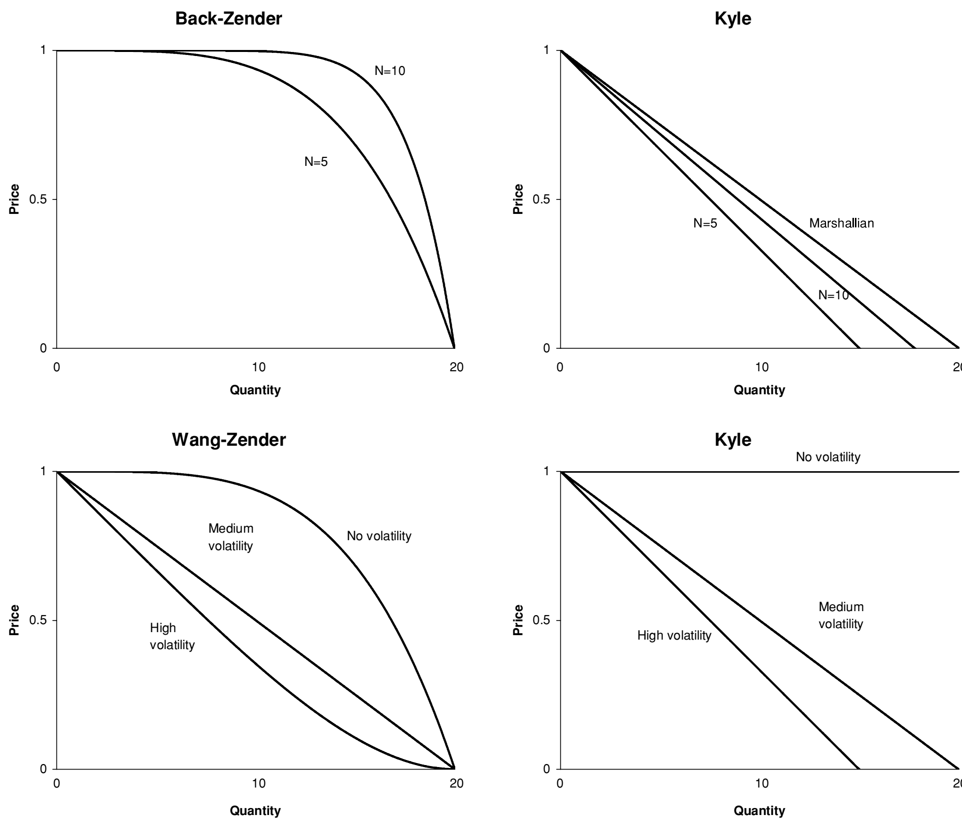

Figure 2. Effect of varying the number of bidders and volatility on individual demand functions. Common parameters are $\bar{v} = 1$ and $r = 0$. Top left: Back and Zender (1993) (risk-neutral bidders), $a = 20$. Top right: Kyle (1989) (risk-averse bidders), $\rho\sigma^2 = 0.05$. Bottom left: Wang and Zender (2002) (risk-averse bidders), $a = 20$, $\sigma = 0$ (no volatility), $\rho\sigma^2 = 0.0375$ (medium volatility) or $\rho\sigma^2 = 0.05$ (high volatility). Bottom right: Kyle (1989) (risk-averse bidders), $N = 5$ and volatility as for Wang–Zender.

bidders' equilibrium demand curves shift out. In Kyle's equilibrium, this translates into increased total demand per bidder. In Wang and Zender's equilibrium, demand curves can be convex when volatility is high, but become increasingly concave as volatility falls. Intuitively, when volatility is high, risk-bearing concerns dominate; when volatility is low, market power concerns dominate. To summarize, Figure 2 shows that the effect of decreasing volatility is similar to that of increasing the number of bidders.

D. Empirical Implications

In this section, we derive testable implications from the models presented above. While the theory assumes that bidders submit "smooth" demand schedules, in practice bidders submit collections of price–quantity pairs, implying that observed demand schedules are step-functions. The theoretical demand schedules should therefore be viewed only as approximations. Furthermore, in our sample, there is variation in auction size and $\bar{v}$ from auction to auction and there is also variation in the number of bids submitted by individual bidders within an auction. Instead of trying to fit discrete empirical demand schedules to the smooth theoretical schedules, our approach is to compute a number of summary statistics of the predicted demand schedules and auction performance, which are straightforward to compare with what we see in the data.

We look at eight measures of bidder behavior and auction performance. The first is the discount, that is, the difference between the expected secondary market price and the quantity-weighted average price of a bidder's demand schedule. Formally, we define the discount of demand schedule $q(p)$ to be

$$\mathit{Discount} = \bar{v} - \bar{p} = \bar{v} - \frac{1}{q(r)} \int_0^{q(r)} p(x)\,dx, \tag{11}$$

where $p(x)$ is the inverse demand schedule, $r \geq 0$ is the reservation price of the seller, and $\bar{p}$ is the quantity-weighted average price along the inverse demand schedule for prices at or above the seller's reservation price. Note that $\bar{p}$ is defined by the last term in (11). This is the appropriate definition since $q(r)$, being the demand at the reservation price, is also the total demand of a bidder who uses $q(p)$. The discount is similar to, but not the same as, underpricing, defined as the difference between the secondary market price and the auction stop-out price

$$\mathit{Underpricing} = \bar{v} - p_0. \tag{12}$$

Note that if the reservation price is binding, then underpricing is simply $\bar{v} - r$.

The next three summary statistics are the standard deviation, skewness, and kurtosis of the inverse demand schedule. The standard deviation of bids along the schedule is

$$\mathit{Standard\ deviation} \equiv \eta = \sqrt{\frac{1}{q(r)} \int_0^{q(r)} (p(x) - \bar{p})^2\,dx}. \tag{13}$$

The formulas for skewness and kurtosis are, respectively,

$$\mathit{Skewness} = \frac{1}{\eta^3 q(r)} \int_0^{q(r)} (p(x) - \bar{p})^3\,dx, \tag{14}$$

and

$$\mathit{Kurtosis} = \frac{1}{\eta^4 q(r)} \int_0^{q(r)} (p(x) - \bar{p})^4\,dx. \tag{15}$$

The sixth measure is total quantity demanded per bidder, $q(r)$, and the seventh is award concentration (see below). Finally, we also look at the standardized discount, defined as the discount divided by the standard deviation.

Table II

Measures of Bidder Behavior and Auction Performance

Back and Zender (1993) column uses (2); model based on risk neutrality and strategic behavior. Kyle (1989) column uses (5); model based on CARA utility, normal distribution, and strategic behavior. Marshallian column uses (7); model based on CARA utility, normal distribution, and competitive behavior. The expression for underpricing assumes that the reservation price is not binding. Otherwise, underpricing equals $\bar{v} - r$. Award concentration is the modified Herfindahl index (17).

	Back and Zender Market Power, Risk Neutral	Kyle Market Power, Risk Averse	Marshallian Competitive Risk Averse
Discount	$\frac{\bar{v}-r}{N}$	$\frac{\bar{v}-r}{2}$	$\frac{\bar{v}-r}{2}$
Standardized discount	$\frac{\sqrt{2N-1}}{N-1}$	$\sqrt{3}$	$\sqrt{3}$
Underpricing	$\bar{v}\left(\frac{Q}{aN}\right)^{N-1}$	$\frac{N-1}{N-2}\frac{\rho\sigma^2 Q}{N}$	$\frac{\rho\sigma^2 Q}{N}$
Standard deviation	$\frac{(N-1)(\bar{v}-r)}{N\sqrt{2N-1}}$	$\frac{\bar{v}-r}{2\sqrt{3}}$	$\frac{\bar{v}-r}{2\sqrt{3}}$
Skewness	$-\frac{2(N-2)\sqrt{2N-1}}{3N-2}$	0	0
Kurtosis	$\frac{3(2N-1)(6-5N+2N^2)}{(4N-3)(3N-2)}$	1.8	1.8
Demand per bidder	$a\left(1-\frac{r}{\bar{v}}\right)^{\frac{1}{N-1}}$	$\frac{N-2}{N-1}\frac{\bar{v}-r}{\rho\sigma^2}$	$\frac{\bar{v}-r}{\rho\sigma^2}$
Award concentration	1	1	1

Table II summarizes the predicted values of these eight statistics for (1) Back and Zender's supply-uncertainty robust equilibrium, (2) Kyle's linear equilibrium, and (3) the corresponding Marshallian demand schedules.[10] Statistics for Wang and Zender's general solution to Kyle's model are so complex that they do not fit in the table.[11]

For the Back and Zender equilibrium, Table II reveals the striking result that the unknown parameter a does not figure in the expressions for the discounts or any of the higher order moments. Furthermore, skewness, kurtosis, and the standardized discount depend only on N. Hence these predictions are valid in a cross section of auctions, even though a, $\bar{v}$, and r may vary from auction to auction. The surprising result that a cancels out may be explained by the fact that the first-order condition of a bidder's price–quantity tradeoff must be satisfied at every point along a supply-uncertainty robust demand function. The concavity of the demand function seen in Figure 2 can be seen in Table II to translate into a negative skewness for the bid distribution. Moreover, taking

[10] Note that the theoretically smallest possible equilibrium stop-out price is $p_{\min} \equiv p(Q_{\max}/N)$, where $p(q)$ is given by, for example, equation (2). So, for $q > Q_{\max}/N$, the functional form of $p(q)$ is irrelevant and arbitrary. The formulas in Table II ignore any such irrelevant demand. One can view r in the formulas as being the maximum of the actual reservation price and $p_{\min}$; r can also be viewed as the lowest price for which a bidder has specified his demand function. When we compute the empirical counterparts to these formulas in Section III, we do not omit any bids because there is no reason to expect that bidders submit bids that have no chance of being awarded.

[11] Exact expressions are reported in an earlier working paper and are available from the authors upon request.

the derivative of the expression for skewness, we see that skewness gets more negative as the number of bidders increases. This is a robust implication of the equilibrium that holds even if the unknown parameters a, $\bar{v}$, and r were to vary systematically with N.

In some of the other comparative statics, which we can compute from Table II for the Back and Zender equilibrium, a and r do not drop out. For example, the discount and the standard deviation decrease with the number of bidders, keeping r fixed. The same holds for underpricing, if we also fix a and assume that the reservation price is not binding. Given a and r, quantity demanded increases with the number of bidders. In short, the arrival of additional bidders induces more aggressive bidding, as a result of diminishing market power. This assumes that r and a do not vary with N in such a way as to offset this effect.

The Kyle equilibrium and its competitive counterpart offer the surprising results that the discount and the higher order moments do not depend on volatility, even though bidders are risk averse. The action is in the quantity demanded. In Kyle's equilibrium, quantity demanded increases with N and decreases with volatility. This quantity effect is a direct consequence of Kyle imposing linearity; this restriction implicitly assumes that the parameter a increases with N and decreases with volatility, as seen in the expression for a at the end of Section II.B.2. The discount is insensitive to volatility because the reduction in demand is precisely such that total risk, $\sigma^2 a$, is kept constant. The quantity effect is also the reason why underpricing is decreasing in N and increasing in volatility. There are three parameter-free tests: the standardized discount, skewness, and kurtosis are constants as a result of linearity. The Marshallian demand function shares all of the Kyle equilibrium's predictions except for the sensitivity to N.

Volatility has a broader impact in Wang and Zender's equilibrium, (10), than in the other equilibria. As seen in Figure 2, as volatility increases, the Wang and Zender demand function becomes less concave and eventually turns convex. So skewness increases with volatility. Keeping the parameter a constant, the discount is also increasing in volatility, since total risk increases. Put in terms of the figure, more weight is placed on lower prices as volatility rises and so the discount falls. The effect of an increase in the number of bidders is qualitatively along the same lines as in Back and Zender, (2). In Tables V and VI in Section III, we present a comprehensive list of the comparative statics of the three market power equilibria and compare these to the empirical comparative statics from regressions of the summary statistics on a set of explanatory variables.

Finally, we examine award concentration. With symmetric bidders and no private information, each bidder receives an equal share of the awards. In the Back and Zender model, for example, bidder i's share, θ_i, equals his demand, (1), evaluated at the stop-out price, (3), divided by the total auction awards, Q. That is, $\theta_i = \frac{q_i(p_0)}{Q} = \frac{1}{N}$. The Herfindahl index is then

$$H \equiv \sum_{i=1}^{N} \theta_i^2 = \frac{1}{N}. \tag{16}$$

Since the number of bidders varies over time in our sample, the Herfindahl index may give the wrong impression of award concentration. For example, if there are five bidders and one bidder gets all the awards, the Herfindahl index equals 1, which is also the case when one bidder gets all the awards in an auction with 10 bidders. Intuitively, the latter case involves more award concentration relative to the benchmark of equal awards to all bidders. To capture this, we employ a modified Herfindahl index

$$H^* = H \times N. \tag{17}$$

This measure equals 1 if bidders submit identical demand schedules, as in the models reviewed above. It is N if one bidder obtains all the awards.

III. Empirical Analysis of Bidder Behavior

This section examines the extent to which the models reviewed above are consistent with observed bidder behavior and auction performance. We run regressions to examine how the six endogenous intrabidder statistics (discount, standardized discount, standard deviation, skewness, kurtosis, and quantity demanded) and the two endogenous auction statistics (underpricing and award concentration) vary with three explanatory variables: volatility, number of bidders (primary dealers), and expected auction size. We also carry out a detailed examination of the nonlinearities in bidders' demand schedules.

A. Descriptive Statistics

Table III provides auction day summary statistics of the exogenous variables in Panel A and the endogenous variables in Panels B–D. The 1,702 demand schedules are submitted in 206 auctions on 175 auction days. For each auction, we compute the equally weighted average of all variables and, when two auctions are held simultaneously, their equally weighted average. This is a conservative way to eliminate correlations among the error terms. Hence, we treat each auction day average as an independent observation.

Panel A reports on volatility and the number of bidders. Volatility is measured as the daily standard deviation of bond returns imposing an ARCH(2) structure (see Appendix B). Daily volatility averages 0.346%. The average number of bidders equals 8.1 and varies from 5 to 10, as shown in Figure 1.

Panel B reports on the two discount measures and underpricing. The estimation procedure for the discount starts by observing that the demand schedule submitted by bidder i in auction j can be represented by the set $\{(p_{ijk}, q_{ijk})\}_{k=1}^{m}$ where m is his number of bids.[12] The quantity-weighted average price of these bids is $p_{ij} = \sum_k w_{ijk} p_{ijk}$, where $w_{ijk} = q_{ijk} / \sum_k q_{ijk}$. The empirical intrabidder discount which corresponds to (11) is then

[12] The number of bids, m, may vary with i and j. For the sake of readability, we have suppressed this dependence in the notation.

Table III

Descriptive Statistics

Means of variables across auction days. Volatility is the conditional standard deviation of daily returns from an ARCH(2) model. The location and dispersion variables are percentage of face value, the quantity variables are millions of markkas of face value, and s.e. is the standard error of the mean. Bidder-level variables (discount, standardized discount, standard deviation, skewness, kurtosis, demand per bidder): For each auction, we first compute the quantity-weighted average for each individual bidder's demand schedule, then the equally weighted average across bidders, and finally the average for the auctions on the same day. The discount is the difference between the secondary market price and the quantity-weighted average bid price. The standardized discount is the discount divided by the standard deviation. Auction-level variables: average across the auctions held on the same day. Underpricing is the difference between the secondary market price and the auction stop-out price. Award concentration is measured by the modified Herfindahl index (17).

Variable	Mean	Std	S.E.	Min	Max	No. Obs.
A: Exogenous						
Volatility	0.346	0.157	0.012	0.110	1.115	175
Number bidders	8.1	2.0	0.1	5	10	175
B: Location						
Discount	0.081	0.153	0.012	−0.397	0.920	159
Standardized discount	0.354	0.764	0.061	−2.017	4.581	159
Underpricing	0.041	0.144	0.012	−0.783	0.420	156
C: Dispersion						
Standard deviation	0.065	0.049	0.004	0.003	0.279	175
Skewness	−0.009	0.428	0.032	−1.623	0.888	175
Kurtosis	2.907	1.547	0.117	1.000	11.184	175
D: Quantity						
Demand per bidder	235	194	15	16	1,390	175
Aggregate demand	2040	1952	148	80	13,903	175
Auction size	1179	850	64	0	4,000	175
Award concentration	2.519	1.258	0.096	1.007	9.000	172

$$DISC_{ij} = P_j - p_{ij}, \tag{18}$$

where P_j is the secondary market price of the underlying security at the time of the auction (see Sec. I.C). For each auction, we compute the equally weighted average of all intrabidder discounts, (18), and then take the equally weighted average across the auctions held on the same day. We also measure the standardized discount as the intrabidder discount (11) divided by the quantity-weighted standard deviation of the bids in the demand schedule.

$$STD_{ij} = \sqrt{\sum_{k=1}^{m} w_{ijk}(p_{ijk} - p_{ij})^2}, \tag{19}$$

That is, the standardized discount equals $DISC_{ij}/STD_{ij}$. It is not defined for one-bid demand schedules. Finally, letting p_{0j} denote the stop-out price in auction j, our empirical proxy for underpricing, (12), is

$$UNDERP_j = P_j - p_{0j}. \tag{20}$$

Secondary market prices are available for 159 auction days, but underpricing can only be estimated in 156 cases since three auctions were cancelled. We see in Panel B that the average discount is positive, 0.081%, and that the treasury securities are underpriced by 0.041% of face value, on average. The discount is significantly different from zero with a t-statistic of 6.7, and underpricing with a t-statistic of 3.4. While the point estimates are small relative to what could occur under the market power theory, they are nevertheless consistent with it since discounts and underpricing depend on the arbitrary parameter a. The standard deviation of the discount is large, 0.153% of face value, and many bids are submitted above the secondary market price. For example, on one auction day the average bid is 0.414% of face value above the secondary market price. In Section V, we compare these estimates with ones from other treasury auction markets. Finally, the average standardized discount is 0.354, which is smaller than the theoretical values implied by Back and Zender (0.750 for $N = 5$ and 0.484 for $N = 10$) and Kyle (1.732).

Panel C reports on the three intrabidder dispersion measures. The estimation procedure follows that for the discount. The empirical proxy for the quantity-weighted standard deviation (13) is given by (19). The empirical proxies for quantity-weighted skewness (14) and kurtosis (15) are, respectively,[13]

$$SKEW_{ij} = \frac{1}{STD_{ij}^3}\left[\sum_{k=1}^{m} w_{ijk}(p_{ijk} - p_{ij})^3\right] \tag{21}$$

and

$$KURT_{ij} = \frac{1}{STD_{ij}^4}\left[\sum_{k=1}^{m} w_{ijk}(p_{ijk} - p_{ij})^4\right]. \tag{22}$$

We can see in Panel C that the average intrabidder standard deviation is about one-fifth of daily volatility. Average skewness is −0.009, which is not statistically different from 0 and therefore consistent with Kyle or any other linear model. However, there is strong evidence against linearity at the individual bidder level. In the pooled sample of individual demand schedules, intrabidder skewness varies from −8.5 to 7.5, with a standard deviation of 1.17. Average skewness also varies widely across auctions. Further evidence against linearity is provided by the average kurtosis of 2.907, which exceeds 1.8 with a t-statistic of 9.5.

Finally, Panel D looks at four quantity measures. The first row shows that the average quantity demanded per bidder per auction is 235 million markkas

[13] For one-bid demand schedules, we set skewness equal to 0 and kurtosis to 1. The rationale is as follows: a single bid can be regarded as the limit as c goes to 0 of two bids of identical sizes at prices $b + c$ and $b - c$. The standard deviation is c, the third moment is 0, and the fourth moment c^4. Hence, skewness is 0 and kurtosis 1. In the limit, as c goes to 0, skewness remains 0 and kurtosis 1.

of face value. The second row shows that aggregate auction demand averages to about two billion, and the third row that the average realized auction size, that is, quantity sold, is about 1.2 billion. There is substantial variation in all measures across auctions. Auction size is zero in three auctions when the Treasury rejected all bids. In the last row, we see that the modified Herfindahl index, (17), averages to 2.5. Thus, bidders do not receive identical awards. The modified Herfindahl index with respect to quantity demanded averages to 1.9 with a standard error of 0.052, showing that awards are more concentrated than demand.

B. Determinants of Bidder Behavior and Auction Performance

In this section, we regress the bidding and auction performance variables on the explanatory variables. The results are in Table IV. One of the regressors is the expected auction size, since this is necessary to examine the hypothesis that bidders are risk averse. While it is clear that expected auction sizes are linked to the Treasury's financing needs, a problem for us is that auction sizes were not preannounced—the Treasury only announced maximum auction sizes after 1998 and never announced minimum auction sizes. Taking the point of view of a bidder, we therefore estimate the expected auction size as the average of the realized sizes of the last three auctions. While this may be a fairly rough estimate, the major empirical results are robust to various alternative specifications, for example, forecasting the auction size using the parameters from the size regression reported below. The regressions in Panels A and B are weighted with volatility. The first three regressions in Panel C are adjusted for first-order autocorrelation using the Cochrane–Orcutt transformation. The award concentration regression is estimated with ordinary least squares.

The overall impression from the regressions in Table IV is that only volatility affects the pricing variables, while all three regressors influence the quantity demanded and sold. Specifically, only volatility is statistically significant in the discount, underpricing, and standard deviation regressions. In contrast, volatility is not significant in the skewness and kurtosis regressions, while expected auction size is. The number of bidders has a significant impact on skewness, but not on kurtosis. None of the regressors are significant in the standardized discount and award concentration regressions. Below we look more closely at the individual regressions and discuss both where the equilibria presented in Section II succeed and where they fail.

Panel A shows that discounts and underpricing increase significantly, both statistically and economically, with volatility. We see that 1 standard deviation increase in volatility (0.157%) raises the discount by 0.050% of face value, which is of the same order of magnitude as the average discount of 0.081% (Table III). It also raises underpricing by 0.034% of face value, which is close to the average sample underpricing of 0.041%. In contrast, the number of bidders has no impact on discounts and underpricing. This is hard to reconcile with the market power theory, since market power should diminish with the

Table IV

Determinants of Bidder Behavior and Auction Performance

Each row is a regression on volatility, the number of bidders, and expected auction size (moving average of previous three realized auction sizes and measured in billions of markkas). The other variables are explained in Table III. The location and dispersion variables are percentage of face value. The quantity variables are millions of markkas of face value; *t*-statistics are in parentheses below. The regressions in Panels A and B are estimated with weighted least squares using volatility as weight. The first three regressions in Panel C are corrected for autocorrelation using the Cochrane–Orcutt transformation, and the regression on award concentration is estimated with ordinary least squares.

Dependent Variable	Constant	Volatility	Number Bidders	Expected Size	R^2	No. Obs.
A: Location						
Discount	−0.037	0.318	0.003	−0.013	0.096	159
	(−0.7)	(3.8)[a]	(0.5)	(−1.0)		
Standardized discount	0.507	0.713	−0.052	0.028	0.041	159
	(1.6)	(1.5)	(−1.5)	(0.4)		
Underpricing	−0.003	0.215	−0.003	−0.009	0.054	156
	(−0.0)	(2.4)[a]	(−0.4)	(−0.6)		
B: Dispersion						
Standard deviation	0.038	0.161	−0.003	−0.019	0.222	175
	(2.2)[a]	(5.9)[a]	(−1.5)	(−0.4)		
Skewness	−0.678	−0.005	0.068	0.101	0.172	175
	(−3.6)[a]	(−0.0)	(3.2)[a]	(2.0)[a]		
Kurtosis	2.374	0.436	−0.021	0.543	0.057	175
	(3.2)[a]	(0.4)	(−0.2)	(2.7)[a]		
C: Quantity						
Demand per bidder	46	−227	21	0.702	0.296	175
	(0.6)	(−2.9)[a]	(2.5)[a]	(2.9)[a]		
Aggregate demand	−380	−2089	289	0.713	0.399	175
	(−0.5)	(−2.9)[a]	(2.9)[a]	(2.7)[a]		
Auction size	−61	−845	134	0.397	0.419	175
	(−0.2)	(−2.6)[a]	(3.7)[a]	(4.0)[a]		
Award concentration	1.580	0.388	0.109	−0.063	0.021	172
	(2.9)[a]	(0.6)	(1.6)	(−0.3)		

[a]Statistical significance 5% or better.

number of bidders. More precisely, the result on the discount is inconsistent with Back and Zender's equilibrium (2). It is also inconsistent with Wang and Zender, (10), when keeping the parameter a fixed, but consistent with Kyle, (5), where a implicitly varies with N so that discounts do not respond to N. The finding on underpricing is inconsistent with all three models. This may be a consequence of how the Treasury sets the stop-out price, which we study in more detail in Section IV. Finally, the standardized discount is insignificantly related to all three explanatory variables. This is consistent with Kyle, who predicts that the standardized discount is a constant, but inconsistent with Back and Zender, who predict that the standardized discount decreases with the number of bidders.

Panel B contains the regressions involving the three intrabidder dispersion measures. The skewness regression is of particular interest, since we saw in Section II that market power may manifest itself through skewness. Indeed, this is the only nonquantity regression in which the number of bidders has a significant impact. Skewness increases by 0.068 for each extra bidder in the auction and increases by 0.101 for each billion in expected auction size. Volatility has no effect. The systematic variation in skewness as the number of bidders changes suggests that bidders employ nonlinear bidding strategies in response to increased competition. What is really striking here, however, is the sign of the coefficient. It is the opposite of the negative effect predicted by Back and Zender's and Wang and Zender's equilibria. It is also inconsistent with Kyle's equilibrium, which predicts that there should be no effect.

Panel B also shows that intrabidder standard deviation increases by a significant 0.0161% of face value per 0.1 percentage point increase in volatility. This stands at odds with Kyle's equilibrium, where each risk-averse bidder responds to uncertainty by reducing quantity demanded, but not by increasing the dispersion of his bids. There is also no role for volatility in Back and Zender's equilibrium, since bidders are risk neutral and do not have private information. However, Wang and Zender's equilibrium could generate this result on standard deviation.

Panel C presents the results of the quantity regressions. In the regression on quantity demanded per bidder, we have normalized the expected auction size regressor by dividing it by the number of bidders. There are three particularly interesting results. First, demand decreases with volatility, which is in line with Kyle's equilibrium. Second, each bidder demands more when there are more bidders. For each new bidder who enters the auction, the typical bidder increases demand by a significant 21 million. This behavior is also consistent with Kyle's equilibrium. Third, bidders demand more when expected auction size increases. The striking observation is that they do so without lowering prices, as can be seen in Panel A. This is hard to reconcile with the hypothesis that bidders are risk averse. Consistent with the interpretation that bidders are risk neutral, bond dealers in the Finnish treasury market have told us that they consider interest rate risk to be relatively small because there are only 30 minutes between the auction and the announcement of the results. To the extent that they are concerned with the risk, they use forward contracts on Finnish and German bonds to hedge the auction bids. Finally, we see that award concentration is insensitive to market conditions, as measured by our three regressors.

The empirical comparative statics from the above regression analysis are summarized in Table V. A "+", "−", or "0" indicates that the regression coefficient is significantly positive, significantly negative, or not significant at the 5% level, respectively. We use boldface to indicate that the predicted sign equals the empirically observed sign. The table also compares the empirical findings with the theoretical comparative statics from the Back and Zender and Kyle models. At the bottom, we report the number of correct and incorrect predictions for each model.

Table V
Summary of Comparative Statics

Comparison of empirical comparative statics from Table IV and theoretical comparative statics from Table II. A "+", "–", or "0" means that the regression coefficient is significantly positive, significantly negative, or not significant at the 5% level. Correct prediction of the theory is marked with boldface, and "?" indicates ambiguous since the parameter a may depend on Q. The asterisk indicates that $r > 0$ is assumed (if $r = 0$, the comparative static would be 0).

	Empirical	Back–Zender	Kyle
A: Discount			
Volatility	+	0	0
Number	0	–	**0**
Expected size	0	?	**0**
B: Standardized discount			
Volatility	0	**0**	**0**
Number	0	–	**0**
Expected size	0	**0**	**0**
C: Underpricing			
Volatility	+	0	+
Number	0	–	–
Expected size	0	?	+
D: Standard deviation			
Volatility	+	0	0
Number	0	–	**0**
Expected size	0	?	**0**
E: Skewness			
Volatility	0	**0**	**0**
Number	+	–	0
Expected size	+	0	0
F: Kurtosis			
Volatility	0	**0**	**0**
Number	0	+	**0**
Expected size	+	0	0
G: Demand per bidder			
Volatility	–	0	–
Number	+	+*	+
Expected size	+	?	0
No. correct	n.a.	**5**	**13**
No. incorrect	n.a.	12	8

Table V shows that Back and Zender's model delivers the right comparative statics in only 5 of the 17 cases where we have unambiguous predictions. Notably, the model fails with respect to the impact of the number of bidders. Most striking is that skewness varies with the number of bidders with the opposite sign in the data and the theory. Kyle's model does better and delivers the right comparative static result in 13 of 21 cases. Kyle predicts correctly that demand per bidder decreases with volatility, but cannot explain the general

Table VI
Numerical Comparative Statics for Wang and Zender (2002)

The variables N and $\rho\sigma^2$ vary so that the inverse demand schedules move from the convex region, across the linear subcase (Kyle (1989)), to the concave region (see Figure 2). The column with zero volatility is the same as Back and Zender (1993), and "yes" ("no") means that the numerical comparative static matches (does not match) the empirical ones from Table IV. The calculations use $\bar{v} = 1$, $r = 0$, $a = 25$, and $Q = 50$.

	Varying Volatility ($N = 7$)				Varying N ($\rho\sigma^2 = 1/30$)			
	Concave $\rho\sigma^2 = 0$	Linear $\rho\sigma^2 = \frac{1}{30}$	Convex $\rho\sigma^2 = \frac{1}{25}$	Match Empirical	Convex $N = 5$	Linear $N = 7$	Concave $N = 10$	Match Empirical
Discount	0.143	0.5	0.571	Yes	0.533	0.5	0.475	No
Standardized discount	0.601	1.732	1.839	No	1.805	1.732	1.699	No
Underpricing	0.0005	0.286	0.343	Yes	0.442	0.286	0.187	No
Standard deviation	0.238	0.289	0.311	Yes	0.295	0.289	0.280	No
Skewness	−1.898	0	0.208	No	0.125	0	−0.076	No
Kurtosis	5.665	1.8	1.731	No	1.770	1.8	1.864	No
No. correct				**3**				**0**

importance of volatility. Kyle also predicts correctly that demand per bidder increases with the number of bidders, which is suggestive of bidders having some market power.

Table VI performs a similar comparative statics exercise for Wang and Zender's equilibrium, (10). Since all statistics depend on a, the table does not report comparative statics with respect to the expected auction size. Because of the complexity of the demand function, unambiguous results do not exist for all statistics. The table therefore reports numerical values for the summary statistics and shows how the numbers change with volatility and N. The last column in each panel also notes whether these changes match the empirical comparative statics.[14] We see that Wang and Zender's equilibrium does fairly well with respect to volatility, but cannot explain the effect on the skewness and kurtosis or the lack of an effect on the standardized discount. Like Back and Zender, however, Wang and Zender's equilibrium fails with respect to the number of bidders, which is the parameter at the heart of the imperfect competition story. Overall, our findings suggest that market power is not a key factor.

[14] We have also derived exact expressions for the derivatives of the various statistics when $r = 0$, which we then have examined numerically and graphically using Mathematica. We have been able to verify that the comparative statics on N and σ indicated in Table VI always hold when N varies between 5 and 10, with the following exceptions: (1) The standardized discount decreases with N except when N goes from 9 to 10 and $a\rho\sigma^2$ is "close" to $\bar{v}/2$; and (2) the standard deviation increases with σ^2 except when $a\rho\sigma^2$ is "small" relative to $\bar{v}$, and decreases with N except when $N = 5$ and $a\rho\sigma^2$ is "close" to $\bar{v}$.

C. Nonlinearity: Skewness, Kurtosis, and Number of Bidders

In this subsection, we take a closer look at the nonlinearity of submitted demand functions and study how skewness and kurtosis vary with the number of bidders. A bidder's set of price–quantity pairs in a generic auction is given by the set $\{(p_k, q_k)\}_{k=1}^{m}$, where m is the number of bids and the bids are ordered by $p_1 > p_2 > \cdots > p_m$. We can think of a demand schedule with $m \geq 2$ as being "discrete linear" if the bidder's marginal demand is the same at every price at which he submits a bid and these prices are spaced equally. We define the standardized difference between adjacent prices to be

$$d_k^* = \frac{p_k - p_{k+1}}{p_1 - p_m} \bigg/ \frac{1}{m-1} \, . \tag{23}$$

There are $m - 1$ price differences. Under a discrete linear strategy, $d_k^* = 1$, skewness is 0, and kurtosis approaches 1.8 from below as N increases.

Table VII reports our findings. Panel A covers the case with few bidders and Panel B covers that with many bidders. Within each panel, the upper subpanel provides the means of d_k^* across all demand schedules with $m = 1, \ldots, 8$ bids. The lower subpanel shows the averages of the intrabidder standard deviation, skewness, kurtosis, and the number of observations. In Panel A, we can see that for all m, the lowest d_k^* exceeds 1. This means that the last price difference is larger than the intermediate price differences. This explains why skewness is negative. However, in Panel B, we can see that for all m, the highest d_k^* exceeds 1, which means that the first price difference is larger than the intermediate price differences. Therefore, skewness turns from negative with few bidders to positive with many bidders. Moreover, this switching of sign is robust to the number of bids in a demand schedule; skewness is consistently negative for 5 to 8 bidders and consistently positive for 9 to 10 bidders, regardless of m. Thus, Table VII corroborates our earlier finding that while skewness is zero on the average, skewness is positively related to the number of bidders. The tendency to submit one bid, which is either much higher or lower than the other bids, also explains why kurtosis is higher than predicted by a discrete linear strategy.

What explains this behavior? One hypothesis is that the dealers find it easier to coordinate their bidding when they are few, which is why skewness is negative. But this would also imply that underpricing and discounts should be larger when the number of bidders is small, which we do not observe. Another possibility is that the observed behavior originates with preselling of securities by primary dealers to customers, with the consequence that dealers who do not cover in the auction may get squeezed in the secondary market. Nyborg and Strebulaev (2004) show that in equilibrium, short bidders submit a bid at a "very high" price for the few units they need to avoid being squeezed. It is possible that this became more of an issue when more dealers entered the market. A third possibility is that the positive skewness reflects customer bids. Dealers have told us that one major institutional investor frequently instructed them to submit "market orders."

Table VII

Dispersion, Skewness, and Kurtosis

Bidder-level demand schedules with $m = 1$ to 8 individual bids. Upper subpanels: Average standardized price differences, (23). Lower subpanels: Average standard deviation, skewness, and kurtosis.

	$m=1$	$m=2$	$m=3$	$m=4$	$m=5$	$m=6$	$m=7$	$m=8$
A: Few Bidders, $5 \le N \le 8$								
d_1^*		1.000	0.935	0.989	1.116	0.829	0.947	1.468
d_2^*			1.065	0.853	0.756	0.786	0.914	0.863
d_3^*				1.156	0.882	0.890	0.820	1.059
d_4^*					1.246	1.292	0.825	0.775
d_5^*						1.207	0.797	0.541
d_6^*							1.700	1.195
d_7^*								1.097
F-test	n.a.	n.a.	7[a]	8[a]	8[a]	4[a]	4[a]	0.3
St. deviation	0.000	0.055	0.078	0.106	0.157	0.138	0.145	0.277
Skewness	0.000	−0.104	−0.249	−0.174	−0.232	−0.521	−0.874	−0.155
Kurtosis	1.000	2.875	3.214	3.489	3.530	2.773	6.340	4.224
No. obs	120	115	66	60	40	12	9	3
B: Many Bidders, $9 \le N \le 10$								
d_1^*		1.000	1.039	1.156	1.402	1.355	1.650	1.744
d_2^*			0.961	0.899	0.831	0.926	1.068	0.885
d_3^*				0.955	0.798	0.923	0.745	0.845
d_4^*					0.970	0.899	0.703	0.678
d_5^*						0.898	0.770	0.835
d_6^*							1.065	0.802
d_7^*								1.208
F-test	n.a.	n.a.	7[a]	19[a]	26[a]	5[a]	7[a]	2[a]
St. deviation	0.000	0.070	0.078	0.099	0.117	0.133	0.175	0.229
Skewness	0.000	0.038	0.105	0.180	0.285	0.347	0.238	0.323
Kurtosis	1.000	3.743	4.081	3.742	4.486	4.693	3.456	4.101
No. obs	385	305	258	171	77	38	17	11

[a]Statistical significance 5% or better.

IV. Strategic Seller Behavior

Our findings thus far suggest that bidders act more competitively than predicted by the market power theory. One possible reason is that the theory assumes that supply is exogenous, while in practice the Finnish Treasury determined the supply after observing the bids. In this section, we explore the strategic behavior of the seller, with an adjunctive view to see how it might affect bidder behavior and market power.

A. Theory

The theory of uniform auctions reviewed in Section II shows that there can be multiple equilibria with very different levels of underpricing, depending on the value of the parameter a. The seller's preferred equilibrium arises when $a = \infty$, which implies that demand curves are infinitely elastic and there is no underpricing ($p_0 = \bar{v}$). This contrasts with the bidders' preferred equilibrium where $a = Q_{\max}/N$. In this case, the seller would be giving away the securities for free if the auction size were $Q_{\max}$. The seller can avoid some bad outcomes by imposing a reservation price $r > 0$. Furthermore, Back and Zender (2001) show that the seller can reduce equilibrium underpricing by choosing the supply ex post to maximize revenue. If the seller behaves this way, in equilibrium bidders submit demand functions such that revenue is maximized at $Q_{\max}$ and the stop-out price is at least[15]

$$p_0 \geq \left(\frac{N-1}{N}\right)\bar{v}. \tag{24}$$

The demand schedule (1) is still equilibrium, but the lower bound on a increases to

$$a \geq \left(\frac{Q_{\max}}{N}\right)N^{\left(\frac{1}{N-1}\right)}. \tag{25}$$

When there are 10 bidders, which is the maximum in our data set, (24) allows underpricing to be anything between 0% and 10%. The observed level of underpricing in our data, 0.041%, falls within this range and is therefore consistent with the theory. The question remains, however, whether it is an ex post revenue maximization rule or something else which lies behind this level of underpricing. It is therefore interesting to explore what the Treasury actually does.

B. Stop-Out Price and Marginal Revenue Maximization

Our approach is motivated by the idea that the seller may wish to choose the stop-out price based on the revenue it will generate. Figure 3 provides an example of the Treasury's typical behavior, using the auction held on October 14, 1993 for a bond maturing in 1996. In this auction, bids were submitted at 10 different price levels, $p_1 < p_2 < \cdots < p_{10}$. The average number of price levels across all auctions is 9.4. For each price level, l, we compute total demand, Q_l, and the total revenue the Treasury could obtain if that price level were chosen as the stop-out price: $\{p_1Q_1, p_2Q_2, \ldots, p_{10}Q_{10}\}$. The figure depicts the normalized revenue curve, where the total revenue for each price level is expressed as a fraction of the maximum revenue which could be generated in the auction, given the submitted bids.

Figure 3 illustrates four important and general facts. First, revenue is maximized at the lowest price level, p_{10}. In 200 of the 206 auctions, this is the

[15] If the seller is willing and able to sell an infinite amount, McAdams (1999) argues that underpricing from market power could be eliminated by the "maximize ex post revenue" rule.

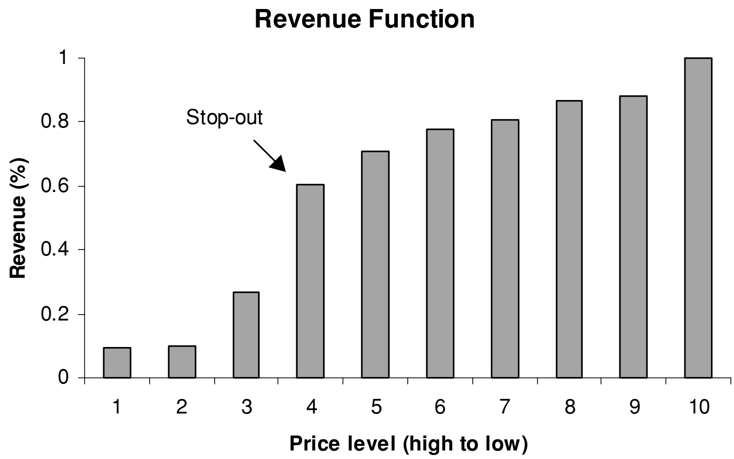

Figure 3. Normalized revenue curve. Auction held on October 14, 1993 of a treasury bond maturing in 1996. There are 10 price levels in this auction, going from high price (level 1) to low price (level 10). Total revenue for each price level is expressed as a fraction of the maximum revenue that could be achieved in the auction. The stop-out price is chosen by the Treasury to be the fourth-highest price level. Total revenue is maximized at the tenth price level, and marginal revenue is maximized at the fourth price level.

case.[16] Second, the revenue-maximizing price level, p_{10}, is not picked as the stop-out price, something which holds true in each and every auction in our sample. Thus, the Treasury does not follow the strategy studied by Back and Zender (2001) and McAdams (1999). Third, marginal revenue is maximized at an internal price level (neither the highest nor the lowest price level). The marginal revenue at level l is defined as the increase in total revenue that could be generated by lowering the stop-out price from level l to $l-1$:[17]

$$MR_l = p_{l-1} Q_{l-1} - p_l Q_l. \tag{26}$$

The maximum marginal revenue occurs at the highest price level in 14 auctions and at the lowest price level in four auctions, but is otherwise, in 188 auctions, located somewhere in the middle. Fourth, the chosen stop-out price coincides with the price at which marginal revenue is maximized.

To examine the generality of the fourth point, we compute the normalized total and marginal revenues for each price level within each auction and compare these with the Treasury's choice of stop-out price. The results are in Table VIII.

[16] In the remaining six auctions, the maximum would be attained at the second-lowest bid (five cases) or the third-lowest bid (one case). These six auctions have in common that the marginal demands at the lowest price are relatively small. Specifically, they are 1 (two auctions), 5, 10 (two auctions), 15, and 60 million markkas.

[17] Note that marginal revenue is usually defined as the extra revenue from increasing the price. We find it intuitive to define marginal revenue in terms of decreasing the price because this captures the idea that the seller is looking at the tradeoff between underpricing (decreasing price) and revenue. Our marginal revenue measure is constructed with the level of underpricing in mind.

Table VIII

Marginal Revenue and Stop-Out Price

For each auction, we identify the price level with the largest marginal revenue, p_0^*, and the two price levels immediately above and below. For each price level, we report the average normalized marginal revenue (normalization by maximum possible revenue in the auction) and the frequency with which the price level is chosen as the stop-out price. This is done in the sample as a whole and in the sample of auctions where bids placed at the stop-out price are rationed. There are 206 auctions, 203 auctions with positive realized supply, and 21 auctions with rationing of bids placed at the stop-out price.

Price Level	Average Normalized Marginal Revenue	Frequency Stop-Out	Frequency among Rationed Auctions	Ave Marg Rev If Rationed
p_{-2}^*	0.085	0.049	0.000	n.a.
p_{-1}^*	0.108	0.034	0.190	0.202
p_0^*	0.360	0.438	0.571	0.457
p_1^*	0.132	0.197	0.048	0.212
p_2^*	0.094	0.133	0.000	n.a.
No. obs	206	203	21	21

In the table, p_0^* denotes the price level with the highest marginal revenue, p_{-1}^* denotes the price level immediately above, p_1^* denotes the price level immediately below, etc. The second column shows the normalized marginal revenue as an average across all auctions in our sample, for five different price levels centered around p_0^*. We see that this average is 36% at the maximal marginal revenue price level, p_0^*. Given that the average number of price levels across auctions is 9.4, this illustrates that a typical auction has a price level where marginal revenue is considerably higher than at other price levels, like price level 4 in Figure 3. One can think of the demand function as exhibiting a kink, or a precipitous drop, at this price level. Alternatively, one can think of the inverse demand function as having a large flat at this price level.[18] The third column contains the key piece of information; namely, how frequently the five price levels are chosen as the stop-out price. We see that p_0^* is chosen in 43.8% of the 203 completed auctions (recall that three auctions in our sample were cancelled). This illustrates the generality of the finding in Figure 3 that the Treasury tends to pick the stop-out price to coincide with the price level where marginal revenue is at its largest.

Another interesting feature of the Treasury's behavior is that it rationed marginal demand at the stop-out price in 21 auctions. The fourth column in Table VIII answers the question as to how many of these rationed auctions coincide with a stop-out price around p_0^*. We see that p_0^* is the stop-out price in 57.1%, or 12, of these auctions. The fifth column tabulates the average normalized marginal revenue at the five price levels for the rationed auctions.

[18] As one might expect, p_0^* tends to be located reasonably close to the quantity-weighted average price of the aggregate demand function. On average, p_0^* exceeds the auction mean by 0.032% of face value.

Comparing these numbers with those in the second column supports the view that rationing tends to happen when marginal demand at the stop-out price is high. For p_0^*, marginal revenue increases from 36% in the sample as a whole (second column) to 45.7% in the sample of rationed auctions (fifth column). This increase is economically large but not statistically significant due to the small number of observations.

The choice of the stop-out price as the price at which marginal revenue is maximized makes intuitive sense when one considers that the Treasury holds a sequence of auctions. What may be surprising is that the Treasury is able to raise the money it needs (to fund the budget deficit) without going below the maximum marginal revenue point more frequently. This could be a result of the Treasury having outside options to borrow elsewhere instead of borrowing expensively in the auction. It could also be that the Treasury's policy induces bidders to be more competitive than suggested by the market power theory. If bidders know that the seller will set the stop-out price where marginal revenue is at the highest, then a single bidder would have an incentive to concentrate demand on that price. However, if all bidders concentrate their demand on the same "consensus" price, rationing will occur. In this case, to avoid rationing, a bidder might find it preferable to concentrate his demand one tick above the others' consensus price. As a result, price competition would ensue and market power would break down. The argument is analogous to Kremer and Nyborg (2004a), who analyze the impact of price and quantity discreteness on market power equilibria. The idea is that the seller's marginal revenue maximization policy may work the same way as increasing the quantity multiple.[19] An important part of the argument is that the Treasury can credibly commit to this policy. It may well be that the fact that the auctions in our sample essentially constitute a repeated game between the Treasury and the primary dealers plays an important role in communicating this policy and making it credible.

C. Underpricing and Auction Size

Within any given auction, the Finnish Treasury faces a price–quantity tradeoff. In this subsection we show that there is no evidence of such a tradeoff *across* auctions. There is no relation between underpricing and realized auction size. When demand is strong, the Treasury sells more securities, and when demand is weak, it holds back supply.

To control for duration (and therefore indirectly for volatility), we work with yields rather than prices.[20] Within each auction, bids are sorted by yield levels that are ordered from the lowest to the highest yield. For each yield level l in

[19] The quantity multiple in Finland is not sufficiently large to eliminate market power equilibria (Kremer and Nyborg (2004a)). However, using the average quantity awarded to bids at the stop-out price (when the stop-out price is the marginal revenue maximizing price) of 495 million markkas as the implicit quantity multiple, Kremer and Nyborg's analysis would predict underpricing less than one tick in our sample. The sample average underpricing is 0.86 ticks.

[20] We have also carried out the analysis in this subsection using prices and reach the same conclusions.

auction j, we compute the difference between the bid yield and the secondary market yield (see Sec. I.C):

$$\text{BID MARKUP}_{lj} = y_{lj} - Y_j. \tag{27}$$

At the chosen stop-out yield, y_{0j}, the markup represents underpricing measured in yield. For each yield level, we compute the aggregate quantity bid up to this yield and standardize by the expected auction size

$$X_{lj} = \frac{Q_{lj} - \bar{Q}_j}{\bar{Q}_j}. \tag{28}$$

For each auction, the locus of points (BID MARKUP$_{lj}$, X_{lj}) essentially sketches out the aggregate (standardized) demand function.

We pool the data across all auctions and estimate the following regression:

$$\text{BID MARKUP}_{lj} = \beta_0 + \beta_1 X_{lj} + \beta_2 X_{lj}^2 + \beta_3 X_{lj}^3 + \varepsilon_{lj}. \tag{29}$$

This provides a characterization of the average aggregate demand schedule. A cubic functional form has been chosen because visual inspection shows that the aggregate demand curve within individual auctions tends to be S-shaped. The independent variable is highly skewed, so we adopt the transformation

$$X_{lj} = \ln\left(1 + \frac{Q_{lj} - \bar{Q}_j}{\bar{Q}_j}\right). \tag{30}$$

The regression coefficients evaluated at the stop-out yield characterize the seller's policy. The tradeoff policy says that $\beta_1 > 0$; the strong no-tradeoff hypothesis says that $\beta_1 = \beta_2 = \beta_3 = 0$.

The regression results are reported in Table IX. In the regression using all yield levels, β_1 is positive. Hence, the aggregate inverse demand schedule (with yields on the y-axis) is upward sloping. Within an auction therefore the Treasury faces a tradeoff between yield and quantity. The estimated values for β_2 and β_3, both significantly negative, tell us that this tradeoff is nonlinear. This contrasts with the regression using only the observations at the stop-out yield.

Table IX

Treasury Policy

Estimation of (29) with ordinary least squares using the markup at each yield level as the dependent variable and the standardized aggregate demand defined by (30) as the independent variable; t-statistics are in parentheses below.

	β_0	β_1	β_2	β_3	R^2	No. Obs
All yield levels	0.0300	0.0178	−0.0035	−0.0007	0.200	1,388
	(15.7)[a]	(9.6)[a]	(−3.1)[a]	(−3.7)[a]		
Stop-out only	0.0064	−0.0005	0.0024	−0.0001	0.014	175
	(2.5)[a]	(−0.1)	(0.8)	(−0.1)		

[a]Statistical significance 5% or better.

Here, only the constant is significantly different from zero.[21] This shows that while the auctioned securities are underpriced on the average, across auctions the Treasury is not trading off underpricing, here measured in yield, and quantity. In other words, the outcome of the repeated game played between the Treasury and bidders is to keep the yield markup (underpricing) unaffected by quantity sold. There may be several reasons for this. First, bidders tend to respond to larger expected auction sizes by increasing quantity demanded without lowering discounts. This helps the Treasury to sell larger quantities without lowering prices. Second, since the Treasury tends to pick the stop-out price where the marginal demand is the largest, it has scope for varying the quantity in an individual auction without changing the price. Our findings imply that the expected auction size does not affect the price level with the largest marginal demand and revenue.

V. Uniform Versus Discriminatory Auctions

Which auction format is revenue superior? The U.S. Treasury switched from discriminatory to uniform auctions in October 1998 after several years of experimentation because of performance improvements.[22] The Finnish Treasury chose the uniform format for its bond auctions because it believed that this would be more conducive to competition—not less!—than discriminatory auctions and thereby ultimately lead to higher auction revenues. This belief was based on interviews with potential bidders and the experience of other countries. However, many countries still employ discriminatory auctions.[23]

Empiricists have approached the revenue question by estimating the level of underpricing. Panels A and B in Table X summarize some of the U.S. and international evidence, respectively. The benchmark in the U.S. studies is the when-issued yield; elsewhere, it is the secondary market price. Our study, like most studies, finds that treasury securities are underpriced in the auctions. The Finnish underpricing of 0.041% of face value translates into 0.78 bp in yield space, which is somewhat larger than in the U.S. Securities sold in uniform auctions appear to be less underpriced than those sold in discriminatory auctions. One must be careful when comparing the studies using prices, however, because the durations of the auctioned securities vary from study to study.[24]

We will compare uniform auctions in Finland with discriminatory auctions in Sweden. For both countries, we have detailed information on auctions in

[21] Lack of cross-section variation in the independent variable does not explain the insignificant coefficients in the stop-out sample, because the standard deviation of X_i is 0.749 in the stop-out sample compared with 1.505 in the full sample.

[22] See Malvey and Archibald (1998), particularly the foreword by the Deputy Secretary of the Treasury at the time, Lawrence Summers.

[23] Countries that regularly use discriminatory auctions to sell treasury securities include the United Kingdom, Italy, Canada, Germany, and Sweden.

[24] For example, in the studies by Bjønnes on Norwegian Treasury auctions, time to maturity ranges from 1 to 11 years in the uniform auctions and from 16 to 365 days in the discriminatory auctions.

Table X
Underpricing in Treasury Auctions

Panel A. Underpricing is the quantity-weighted average yield or rate paid in the auction less the when-issued yield or rate. Panel B. Underpricing is the secondary market price less the quantity-weighted average price paid in the auction. The benchmark column describes what underpricing is measured relative to.

Discriminatory		Uniform				
Underp.	No. Obs	Underp.	No. Obs	Benchmark	Country	Authors
A: Yields and Rates (Basis Points)						
1.3[a]	364	–	–	Bid quote	U.S.	Spindt and Stolz (1992)
0.37[a]	66	–	–	Bid quote	U.S.	Simon (1994)
0.27[a]	76	0.09	15	Transactions	U.S.	Nyborg and Sundaresan (1996)
0.55[a]	66	0.21	44	Bid quotes	U.S.	Malvey and Archibald (1998)
0.61[a]	105	0.40[a]	178	Transactions	U.S.	Goldreich (2003)
B: Prices (Percentage of Face Value)						
0.018[a]	181	−0.003	26	Transactions	Mexico	Umlauf (1993)
0.028	56	–	–	Bid quote	Japan	Hamao and Jegadeesh (1998)
0.020[a]	458	–	–	Bid quote	Sweden	Nyborg et al. (2002)
0.036[a]	68	0.133[a]	34	Midpoint	Norway	Bjønnes (2001, 2002)

[a]Statistical significance 5% or better.

different duration bands. A straight comparison of mean underpricing finds it being lower in Swedish discriminatory auctions (0.020% of face value) than in Finnish uniform auctions (0.041%). However, these numbers are not directly comparable for two reasons. First, the Swedish sample includes treasury bills and bonds, whereas the Finnish sample includes bonds only. Second, the Swedish auctions are benchmarked against pure bid quotes and the Finnish auctions against transaction-adjusted bid quotes (see Sec. I.C). Since transactions often happen within the bid–ask spread, this means that the estimation procedure used for the Swedish data understates the discount and underpricing relative to the Finnish estimates.[25] To deal with the first problem, we break up the data into different duration bands and perform the comparison on this basis. As we shall see, this also makes the second problem less of an issue.

Table XI contains the comparison. Panel A reports the mean discount for both uniform and discriminatory auctions, broken down by seven duration bands as well as for the pooled sample of treasury bonds. Panels B–D do the same for underpricing, standard deviation, and volatility. We see that for each duration band, bidders in Finnish uniform auctions submit their bids at lower discounts and disperse their bids less than in Swedish discriminatory auctions. The pooled sample means are also statistically significantly lower for the uniform auctions. This translates into consistently lower underpricing for uniform

[25] Since the bid–ask spread tends to be much larger in Finland, using the pure bid quote for the Finnish data will not resolve the issue of comparability.

Table XI

Comparison of Uniform and Discriminatory Auctions

Average discount, standard deviation, underpricing, and volatility for treasury bonds across seven duration bands. The variables are explained in Table III. The table compares uniform auctions in Finland with discriminatory auctions in Sweden (Nyborg et al. (2002, Table 4)). "All" refers to the average across duration bands. The *t*-statistic tests the hypotheses that the means in the "All" column are equal in discriminatory and uniform auctions.

Format	Duration (Years) 2	3	4	5	6	7	8	All	*t*-Test Diff.	No. Obs
A: Discount (Percentage of Face Value)										
Discriminatory	0.073	0.189	0.154	0.252	0.478	0.442	0.295	0.306	4.1[a]	93
Uniform	0.036	0.026	0.020	0.059	0.123	0.181	0.062	0.081		159
B: Underpricing (Percentage of Face Value)										
Discriminatory	−0.001	0.061	0.017	0.086	0.204	0.195	0.109	0.120	1.6	93
Uniform	0.019	0.010	0.004	0.043	0.028	0.128	0.067	0.041		156
C: Standard Deviation (Percentage of Face Value)										
Discriminatory	0.040	0.067	0.086	0.122	0.162	0.148	0.149	0.126	7.8[a]	93
Uniform	0.021	0.044	0.050	0.063	0.077	0.095	0.078	0.065		175
D: Volatility (Percentage of Price)										
Discriminatory	0.259	0.334	0.408	0.546	0.600	0.509	0.888	0.496	4.5[a]	93
Uniform	0.174	0.238	0.337	0.314	0.439	0.399	0.361	0.346		175

[a]Statistical significance 5% or better.

auctions, except for 2-year bonds. In the pooled sample of all bonds, the average underpricing is 0.041% of face value in Finland and 0.120% in Sweden. The *t*-statistic for a differences in means test is 1.6, which translates into a significance level of 11% for a two-tailed test and 5.5% for a one-tailed test. Keeping in mind the negative bias in the measured underpricing levels (and discounts) under the Swedish discriminatory auctions, we think this is fairly strong evidence that underpricing is lower in the Finnish sample. This could be due to volatility being lower in the Finnish market (Panel D). However, dividing discounts, underpricing, and standard deviation by volatility, we find that the volatility-adjusted means are also consistently smaller in Finland.[26] This suggests that it is the uniform format rather than the lower volatility that lies behind our findings.

Finally, we compare the regression results in Table IV with those obtained by Nyborg et al. (2002, Table 5) for Swedish discriminatory auctions. The empirical results are similar. In both markets, volatility has a significant positive impact on underpricing, discount, and standard deviation, and has a negative impact

[26] The exceptions are volatility-adjusted underpricing for 2- and 8-year bonds and standard deviation for 8-year bonds.

on quantity demanded. These regression coefficients also have the same order of magnitude in the two studies, as do the means of volatility and the endogenous variables. Furthermore, in both markets, auction size has at most a negligible effect on these variables, with the exception of quantity demanded, which is increasing in auction size.[27] These findings are consistent with the hypothesis that private information and the winner's curse are important factors under either format. The lower underpricing in the uniform auctions is then consistent with the view that uniform auctions result in a lower underpricing because they reduce the winner's curse.

VI. Conclusions

This paper analyzes bidder behavior and underpricing in uniform price treasury auctions with a small number of bidders. We derive and test implications of the theory of uniform price auctions which emphasizes market power. The finding that individual bidders' demand increases when there are more bidders is consistent with the argument that bidders exercise market power. However, the observations that discounts and underpricing are unaffected by the number of bidders (which is exogenous) are not. Moreover, the specific equilibria of Back and Zender (1993), Kyle (1989), and Wang and Zender (2002) cannot explain the observed nonlinearities in bidders' demand schedules. Most problematic for the market power theory, the skewness of individual bid functions is increasing in the number of bidders, as opposed to decreasing as predicted by the theory. Finally, risk bearing does not seem to influence bidder behavior. As auction size increases, bidders willingly purchase larger quantities without lowering the prices at which they bid.

A reason why the market power theory of uniform auctions is rejected may be that the seller acts strategically with respect to the amount sold, rather than being passive as assumed in the theory. We have documented that the Finnish Treasury appears to have a policy that can best be described as one of maximizing marginal revenue. It may well be that this policy creates incentives for bidders to concentrate their demand around a "consensus" price. In turn, this may create competition for marginal units and thereby help break the noncompetitive equilibria along the lines of Kremer and Nyborg's (2004a) analysis of a discretized uniform price auction. The fact that the auctions are held repeatedly may play a role here by serving as a mechanism that communicates the Treasury's policy to the market and makes it credible.

Another possibility is that the Treasury may have outside options to borrow from different sources or to use different mechanisms if it is not happy with the bids it receives in the auction. Furthermore, the theory treats the auction as a one-shot game while the treasury auctions in our data are repeated. It does not seem plausible that the Treasury is willing to tolerate very low prices in the auction without either disciplining primary dealers or taking its business

[27] Nyborg et al. (2002) do not report on skewness and kurtosis and they do not study the impact of variation in the number of bidders.

elsewhere. Such threats can serve to weaken primary dealers' willingness and ability to coordinate on an underpricing equilibrium. A contrasting view is that repetition enhances bidders' market power by facilitating coordination among them, as emphasized in the experimental study by Goswami, Noe, and Rebello (1996) who find that subjects play Back–Zender type equilibria when they are allowed to communicate before the auction, but not otherwise. Weighing these views against each other, our evidence suggests that the Treasury's power to discipline dealers dominates the effect of dealers' enhanced ability to coordinate. Studying multiunit auctions as repeated games between the seller and the buyers seems to be an important direction for future research.

Our findings point to the implementation of a multiunit auction as an important factor in determining performance. Strategic behavior on the part of the seller may overcome apparent deficiencies in the auction design. This may also explain why uniform auctions in the United States, for example, have performed well. While the U.S. Treasury does not give itself the extreme flexibility with respect to determining supply that the Finnish Treasury does, it "reserves the right to accept or refuse to recognize any or all bids."[28] The threat of reducing supply if bids are too low provides the U.S. Treasury with protection against very low prices. Knowing this, dealers may not find it worthwhile to pursue underpricing equilibria. Consistent lowballing by a dealer may also lose him his primary dealer privileges.

Finally, this paper reinforces the findings of many other studies that volatility has significant impact on bidder behavior in treasury auctions. When volatility increases, bidders increase discounts, reduce quantity demanded, and increase the dispersion of their bids. This is the same reaction as in Sweden's discriminatory price treasury auctions (Nyborg et al. (2002)). This is noteworthy because market power should not be a concern in these auctions (Back and Zender (1993)) and there is little evidence that risk aversion is a significant driver of bidder behavior in Sweden either. Our findings on volatility are consistent with the view that bidders have private information and are concerned with the winner's curse. However, if bidders do face the winner's curse, we are left with a puzzle as to why discounts do not increase with the number of bidders.

Appendix A: Equilibria in Kyle's Model

This appendix shows the derivation of the equilibrium demand schedules in Kyle (1989) when bidders do not have private information. The approach follows Kremer and Nyborg (2004a). We suppose initially that the supply, Q, is known, but we shall see that the results are robust to supply uncertainty. When all other bidders use $q(p)$, the "final" bidder's optimization problem can be written (because of CARA utility and normality) as

$$\max_p (v - p)(Q - (N-1)q(p)) - \frac{1}{2}\sigma^2\rho(Q - (N-1)q(p))^2, \qquad \text{(A1)}$$

[28] See ftp.publicdebt.treas.gov/gsr31cfr356.pdf, §356.33 Reservation of rights.

where $(Q - (N-1)q(p))$ is the residual supply. The first-order condition is

$$-(Q - (N-1)q(p)) - (N-1)(\bar{v} - p)q'(p) + \sigma^2\rho(Q - (N-1)q(p))(N-1)q'(p) = 0. \tag{A2}$$

Using symmetry and market clearing, $Nq(p) = Q$, the first-order condition is

$$-q(p) - (N-1)(\bar{v} - p)q'(p) + \sigma^2\rho q(p)(N-1)q'(p) = 0. \tag{A3}$$

This is an ordinary differential equation, which is independent of Q. Therefore, the solution to the differential equation will work for any Q. In other words, we obtain supply-uncertainty robust equilibria. There are many possible solutions. To get Kyle's solution, posit a linear equilibrium: $q(p) = \gamma\bar{v} - \gamma p$. Plug $q'(p) = -\gamma$ into (A3). We get

$$q(p) = \frac{(N-1)(\bar{v} - p)\gamma}{\sigma^2\rho(N-1)\gamma + 1}. \tag{A4}$$

This implies that

$$\frac{(N-1)\gamma}{\sigma^2\rho(N-1)\gamma + 1} = \gamma. \tag{A5}$$

Solving for γ, we obtain

$$\gamma = \frac{N-2}{(N-1)\sigma^2\rho}. \tag{A6}$$

Thus, we get Kyle's solution (4).

The general solution to (A3) is not known, but we can obtain the general solution in inverse form by writing (A3) as follows:

$$p'(q)q - (N-1)[\bar{v} - p(q)] + (N-1)\sigma^2\rho q = 0. \tag{A7}$$

The general solution to (A7) is Wang and Zender's (2002) equilibrium (10), where $a > 0$. Note that the general solution is a polynomial function of order $N - 1$ and therefore for $N > 5$, we are unable to find a general closed-form solution for $q(p)$. (As is well known, Abel's classical theorem shows that there is no general formula for the roots of a polynomial of degree five or higher).

Appendix B: Volatility Estimation

We estimate conditional volatility as an ARCH(2) process of bond returns, which have been calculated from end-of-day bid quotes. The cross-section and time-series data are stacked. Let P_t be the bond price at time t and A be the one-day accrued interest for a coupon bond. We assume that bond returns follow a random walk with constant drift a

$$\frac{P_t - P_{t-1} + A}{P_{t-1}} = a + e_t. \tag{B1}$$

Table BI
Parameters and Standard Errors of the ARCH(2) Process

α_0	α_1	α_2	ϕ_1
−0.0017	0.2959	0.2784	0.0179
(0.0013)	(0.0187)	(0.0182)	(0.0005)

The cross-section and time-series data are pooled. The volatility of the error term is

$$e_t^2 = \alpha_0 + \alpha_1 e_{t-1}^2 + \alpha_2 e_{t-2}^2 + \phi_1 DUR_t + \nu_t. \qquad \text{(B2)}$$

The estimated coefficients and standard errors are provided in Table BI.

When a new security is auctioned, there are no bond prices from the secondary market before the auction. In such cases, we use the prices of the traded treasury bond with duration that most closely mimics the duration of the new treasury bond. When a new treasury bond is auctioned, we use the average winning auction yield to compute duration.

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