Reachability for high-dimensional systems via applied Koopman Operator theory

Keywords. Koopman Operator theory, Lyapunov functions, Safety certificates, Data-driven techniques, Machine learning

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Research Problem Statement. Modern systems are growing in complexity and sophistication, as we push their performance towards their extremes. But oftentimes this comes at the expense of greater risks of failures. For highly safety-critical systems, it is therefore crucial to design control and optimization algorithms that generate provably safe behaviours as well as are robust to external perturbations and uncertainties in the system model and the environment it interacts with. Formal guarantees typically involve verifying safety, which determines whether all trajectories from the initial set avoid entering the unsafe set, and its dual problem, reachability verification, which is a classical problem of determining if at least one trajectory starting from the initial set will reach the target set in finite time [1][2].

Model-based nonlinear control theory provides a wide range of mathematically rigorous and systematic methodologies for reachability verification [3]. However, there are many unresolved challenges in deploying these results in the real world, and these challenges have only grown over the recent years due to rising dimensionality and complexity of our systems. Blending learning-based approaches have shown promise in filling such gaps in traditional control-theoretic approaches [4][5]. Such approaches are especially alluring since an accurate model may not always be available in many real-world scenarios.

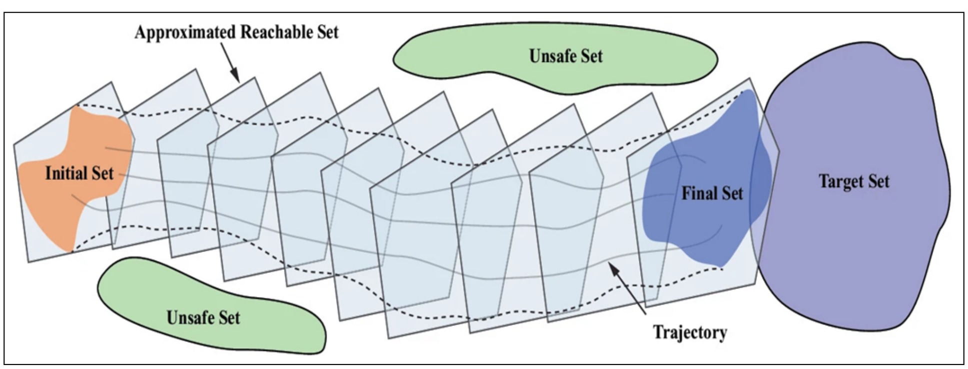

Figure 1: Safety verification based on reachable set computations (https://github.com/ASAG-ISCAS/PyBDR).

Expected workload: If you would like to contribute to this topic, your work will include

1. Getting familiar with literature on learning-based reachability verification, and computational tools.

2. Utilize Koopman Operator theory to develop mathematically rigorous yet practical framework for data-driven control and analysis of robotic systems.

3. Implement high-dimensional multi-agent control algorithms and demonstrate the developed framework to design safe controllers with performance guarantees.

Prerequisite: Good knowledge on control theory, machine learning, and optimisation; Working proficiency with Python and Matlab, basic experience with PyTorch, strong motivation to perform hands-on experiments is a plus.

Type of work: 50% theory, 50% implementation.

References

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