Learning for control and safety certification in robotic systems

Keywords. **Koopman Operator theory, Lyapunov functions, Safety certificates, Data-driven techniques, Machine learning**

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Research Problem Statement. Modern data-driven techniques have rapidly progressed beyond modelling and systems identification, with a growing interest in learning high-level dynamical properties of a system, such as safe-set invariance, reachability, input-to-state stability etc. In particular, developing Lyapunov-based formal guarantees for safety and performance of dynamical systems is an essential step in the design and control of cyber-physical systems. In the absence of a well-known dynamical model, leveraging trajectory data for control and analysis is particularly well-suited. Even for the cases where we have a model, it often takes expert intuition and knowledge to handcraft Lyapunov functions.
In this thesis, we are interested in exploring how one can leverage Koopman Operator theory [1] towards learning Lyapunov functions by efficient utilization of trajectory data, in a more interpretable manner compared to existing literature [2][3]. These computed Lyapunov functions are expected to be verifiable formally and further refined via optimisation tools such as sum-of-squares based semidefinite programming [4], SMT solvers [2] or off-the-shelf neural-net verification tools [5], depending on how the learnt functions are parameterized. Specifically, the thesis will investigate the scalability of this data-driven approach to high-dimensional dynamics [6] where traditional techniques struggle, while focusing on applications in networked and multi-agent control systems.

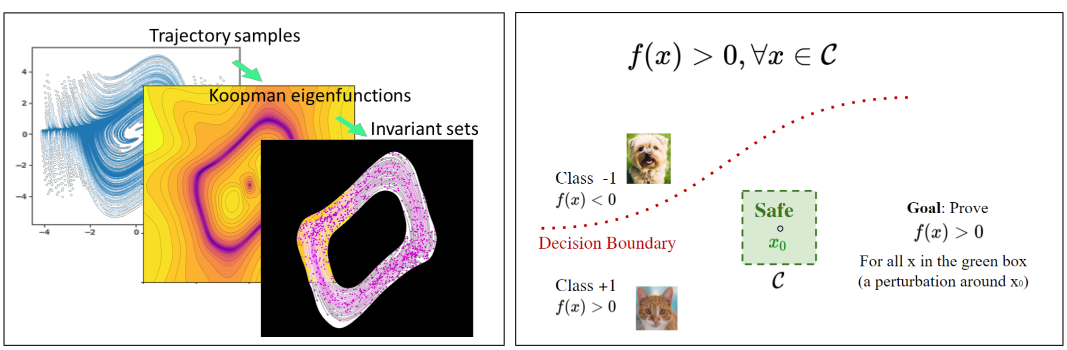

Figure 1: (Left) Pipeline for Koopman-based construction of Neural Lyapunov certificates. (Right) Typical setup for formal verification of neural network input-output properties (https://neural-network-verification.com/).

Expected workload: If you would like to contribute to this topic, your work will include

1. Getting familiar with literature on learning-based Lyapunov safety and performance certificates, and formal verification tools.
2. Utilize Koopman Operator theory to develop mathematically rigorous yet practical framework for data-driven control and analysis of robotic systems.
3. Implement high-dimensional multi-agent control algorithms and demonstrate the developed framework to design safe controllers with performance guarantees.

Prerequisite: Good knowledge on control theory, machine learning, and optimisation; Working proficiency with Python and Matlab, basic experience with PyTorch, strong motivation to perform hands-on experiments is a plus.

Type of work: 50% theory, 50% implementation.

References

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