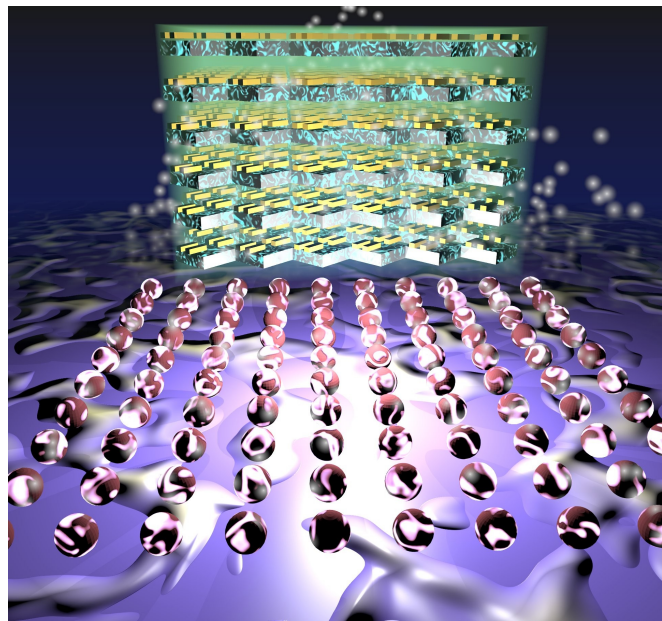


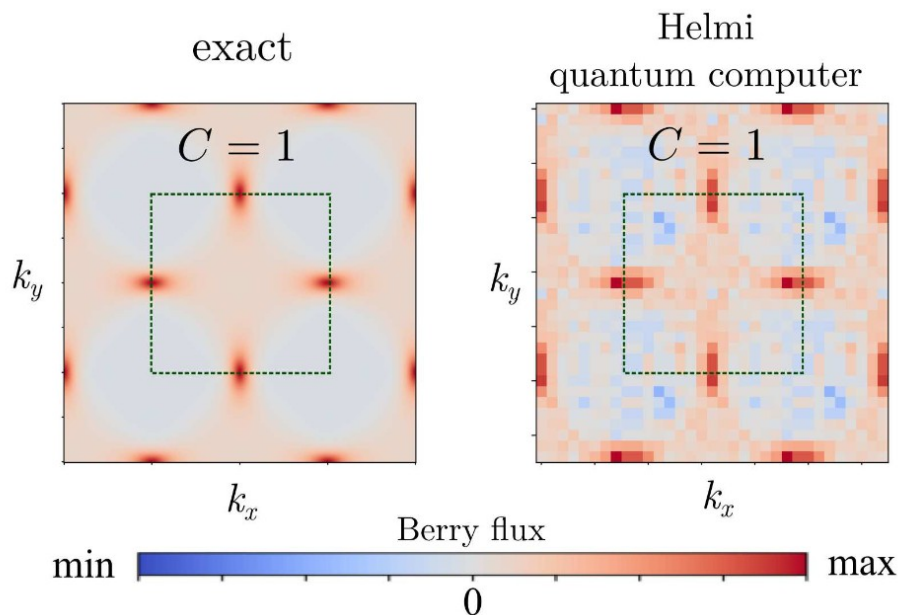
# Solving topological quantum matter on a quantum computer

Jose Lado

*Department of Applied Physics, Aalto University, Finland*

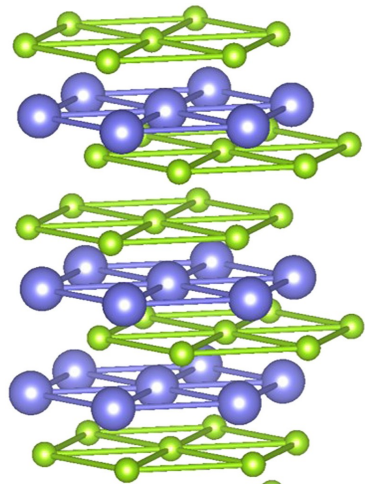


*Phys. Rev. Research 6, 043288 (2024)*



# Emergence in quantum materials

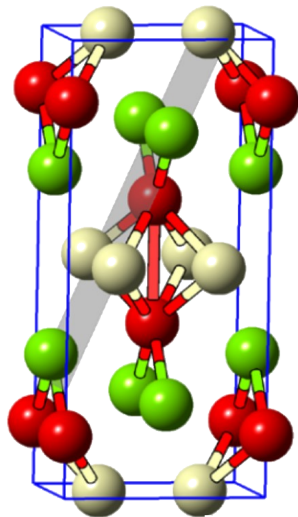
## Topological materials



$\text{Bi}_2\text{Se}_3$

*Science*, 325(5937),  
178-181 (2009)

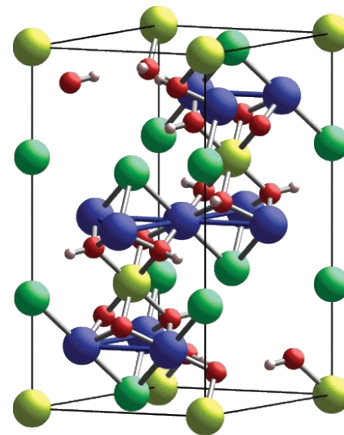
## Unconventional superconductors



$\text{UTe}_2$

*Nature* 579,  
523–527 (2020)

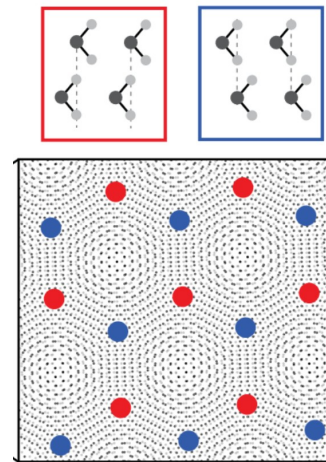
## Quantum spin liquids



$\text{ZnCu}_3(\text{OH})_6\text{Cl}_2$

*Science* 350(6261),  
655-658 (2015)

## Fractional topological matter



Twisted  $\text{MoTe}_2$

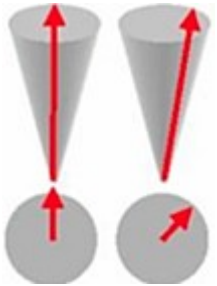
*Nature* 622, 63–68 (2023)

# Quantum excitations in quantum materials

Quantum materials made of electrons (spin =  $1/2$ , charge = 1), protons, neutrons and photons

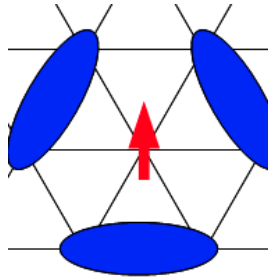
**But in quantum materials, we can have emergent collective new excitations**

Magnons



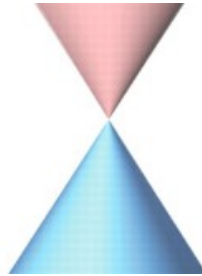
No charge  
Spin = 1

Spinons



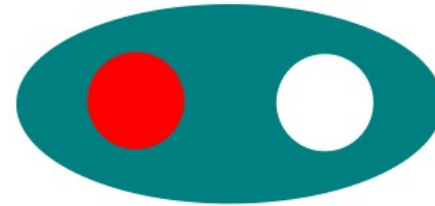
No charge  
Spin =  $1/2$

Dirac fermions



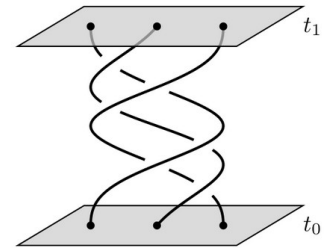
No mass  
Spin =  $1/2$   
Charge = 1

Majorana fermions



No charge  
No spin  
Own antiparticle

Anyons

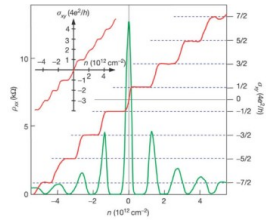


No spin  
Charge =  $1/3$   
Not a boson  
Not a fermion



# Topological quantum matter

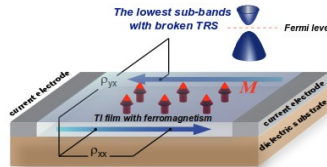
## Quantum Hall effect



graphene

Nature 438, 197-200 (2005)

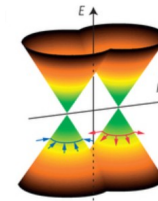
## Quantum Anomalous Hall effect



Science 340.6129 (2013): 167-170

Cr-doped  
(Bi,Sb)<sub>2</sub>Te<sub>3</sub>

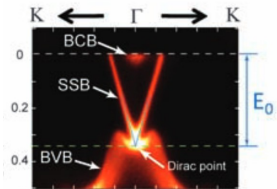
## Weyl semimetal



TaAs

Nature Physics 11, 724–727 (2015)

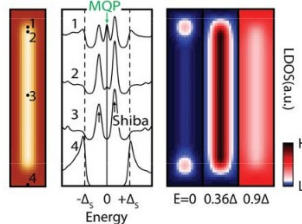
## Quantum Spin Hall effect



Bi<sub>2</sub>Se<sub>3</sub>

Science 325.5937 (2009): 178-181

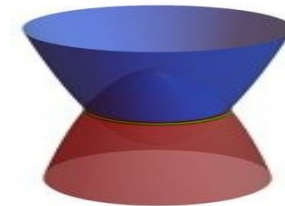
## Topological superconductor



Fe@Pb

Science 346.6209 (2014): 602-607

## Nodal line semimetals



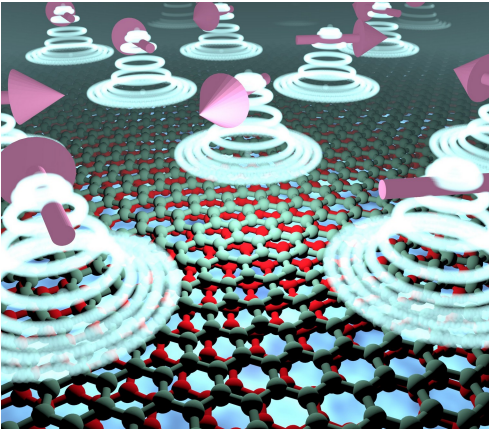
Mg<sub>3</sub>Bi<sub>2</sub>

Advanced Science 6.4 (2019): 1800897

***All these states are described by effective isolated single-particle physics***

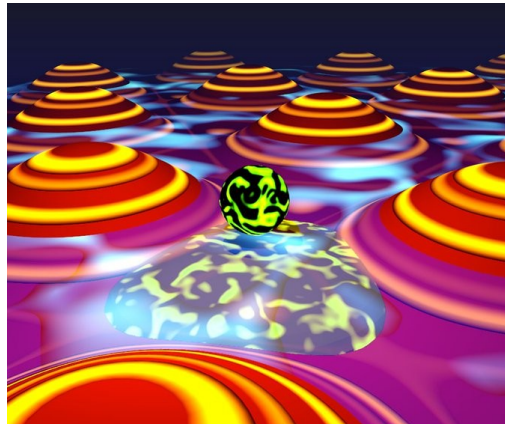
# Many-body topological quantum matter

## Quantum spin liquids



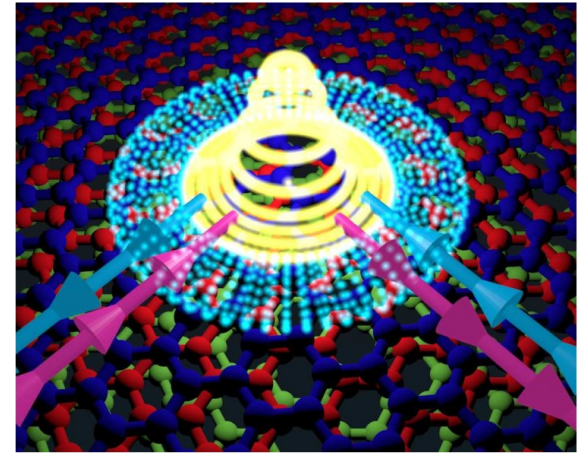
*Reports on Progress in Physics* 80.1 (2016): 016502.

## Fractional Chern insulators



*Rev. Mod. Phys.* 71, S298 (1999)

## Parafermions

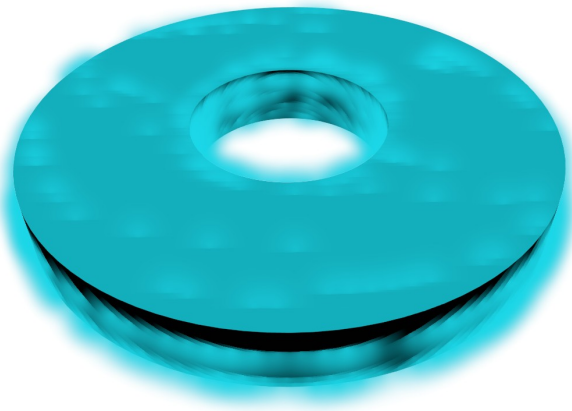


*Annual Review of Condensed Matter Physics* 7, 119-139 (2016)

***Topological many-body quantum matter is an open challenge  
where quantum algorithms will potentially enable solving open challenges***

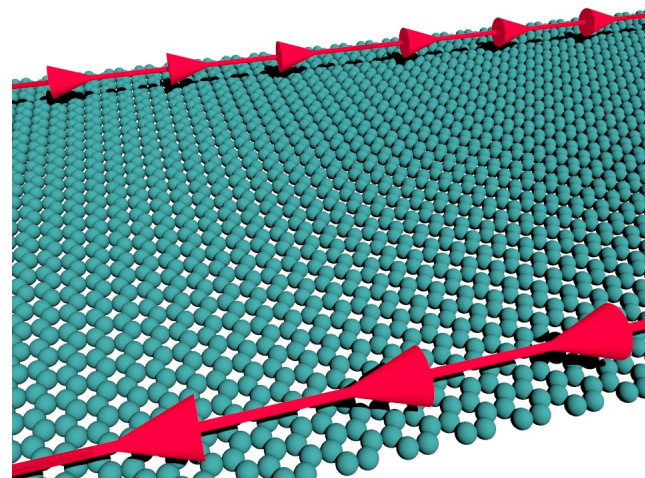
# Macroscopic topological effects

## Superconductivity



$$\Phi = \frac{h}{2e} \quad \text{Quantization of flux}$$

## Chern topological insulators



$$\sigma_{xy} = \nu \frac{e^2}{h} \quad \text{Quantization of conductance}$$

**Macroscopic (topological) quantum phenomena determine fundamental physical constants with the highest precision**



# The computational challenge in topological matter

## Single particle topological matter

$$H = \sum c_i^\dagger c_j$$

$N$  sites/atoms/orbitals

$N$  computational resources

### *How can it be solved*

Exact classical methods

## Many-body topological matter

$$H = \sum c_i^\dagger c_j + V_{ijkl} c_i^\dagger c_j c_k^\dagger c_l$$

$N$  sites/atoms/orbitals

$2^N$  computational resources

### *How can it be solved*

*Exact classical methods (tiny systems)*

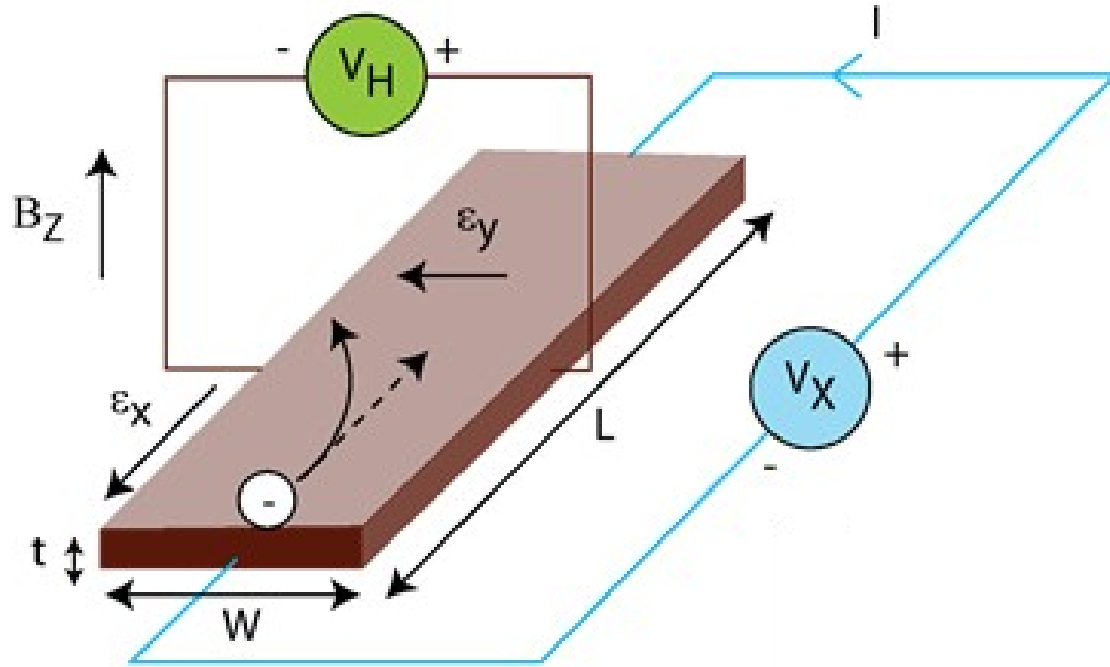
Tensor network classical methods

Quantum computers (?)

# Introduction to topology in quantum materials



# The Hall effect



$$J_x = \sigma_{xy} V_y$$

Hall conductivity

Measure the current perpendicular to a voltage

# The Hall conductivity

The Hall conductivity is obtained as  $\sigma_{xy} = \sum_{\alpha \in occ} \int \Omega_{\alpha} d^2 \mathbf{k}$

**Berry curvature**

$$\Omega_{\alpha} = \partial_{k_x} A_y^{\alpha} - \partial_{k_y} A_x^{\alpha}$$

**Berry connection**

$$A_{\mu}^{\alpha} = i \langle \partial_{k_{\mu}} \Psi_{\alpha} | \Psi_{\alpha} \rangle$$

$$\sigma_{xy} = \sum_{\alpha \in occ} \int \Omega_{\alpha} d^2 \mathbf{k} = \sum_{\alpha} C_{\alpha} = C \longleftarrow \text{Chern number}$$

**The transverse conductivity is a topological invariant**

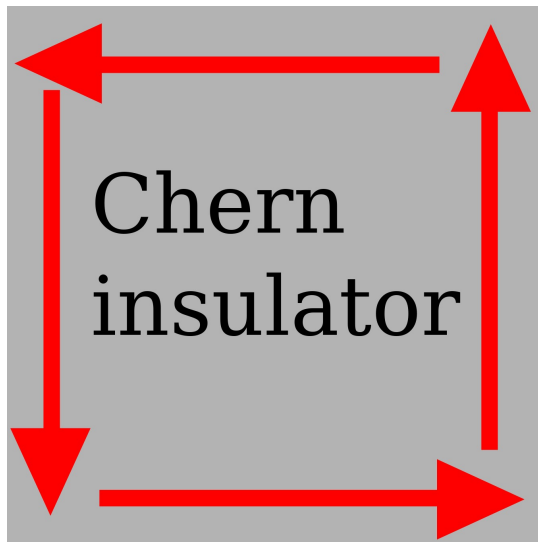
This means, it must take integer values regardless of defects in an insulator

# Hall conductivity in an insulator

$$\sigma_{xy} = \sum_{\alpha \in occ} \int \Omega_{\alpha} d^2 \mathbf{k} = \sum_{\alpha} C_{\alpha} = C$$

***The Chern number is quantized***

$$C_{\alpha} = \int \Omega_{\alpha}(\mathbf{k}) d^2 \mathbf{k} = 0, \pm 1, \pm 2, \dots$$



***An insulator can have a finite  
(and quantized) Hall conductivity***

**This is a simple example of  
a topological state of matter**

# The idea of topological invariants

Holes in a 3d surface



Knots in curves

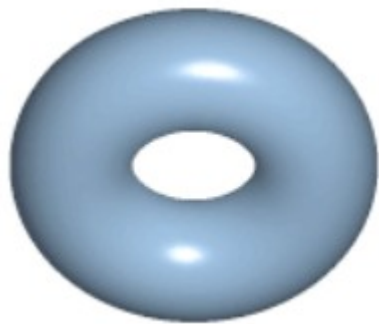


**Topology classifies object that cannot smoothly deformed into one another**



# Topology and holes

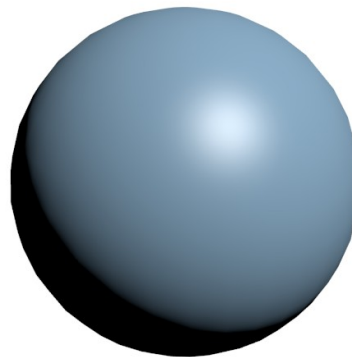
One hole



One hole



Zero holes



“Topologically” equivalent

“Topologically” different

# Topological invariant in a Hamiltonian

We can classify Hamiltonians according to topological invariants

Hamiltonian

$$H(\mathbf{k})$$

Wavefunction

$$|\Psi(\mathbf{k})\rangle$$

$$C = \int_{BZ} \Omega d^2\mathbf{k}$$

Topological invariant  
(Chern number)

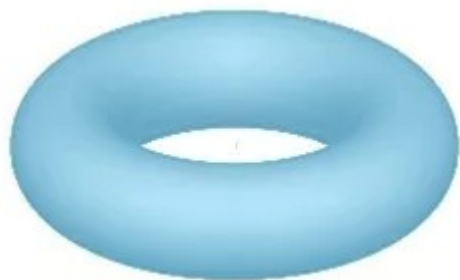
$$\Omega = \nabla \times \mathbf{A}|_z$$

Metric  
(Berry curvature)

# The role of a topological invariant

**Hamiltonians with different topological invariants  
can not be deformed one to another**

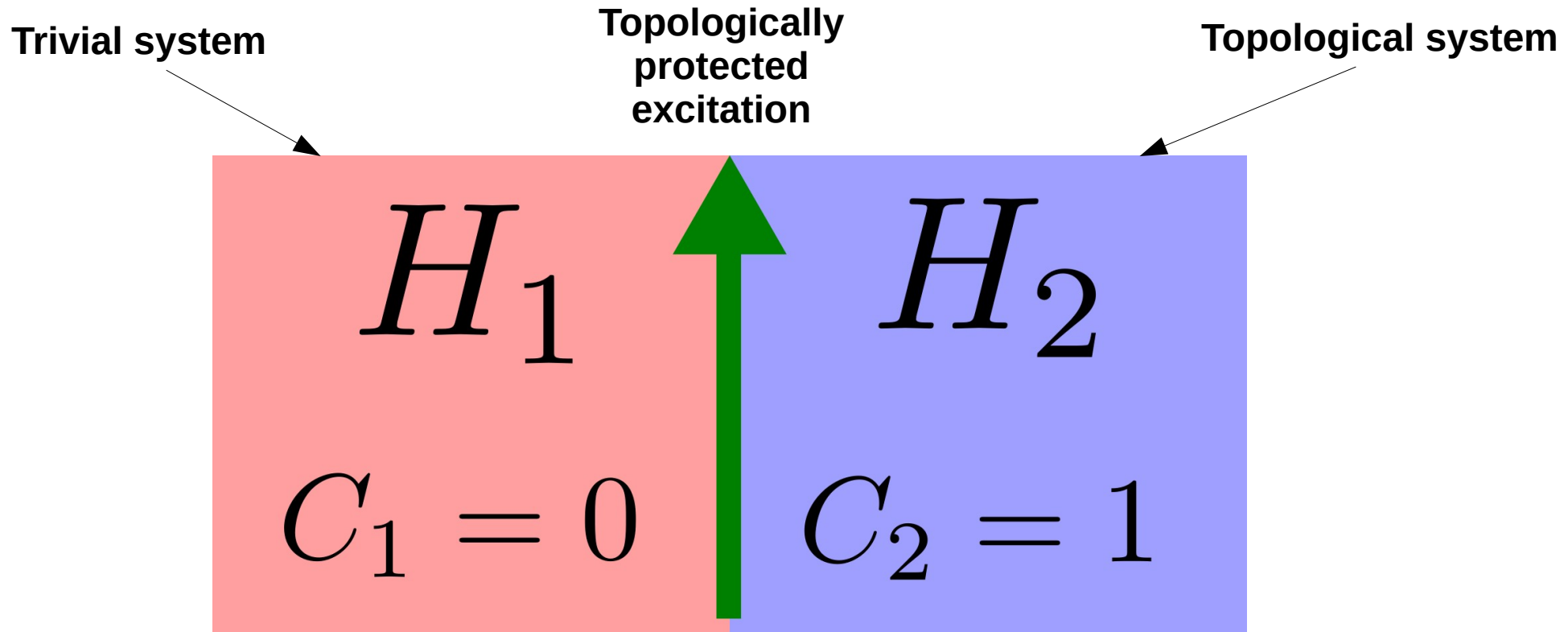
$$C = 1$$



$$C = 2$$



# The consequence of different topological invariants



*Topological excitations appear between topologically different systems*

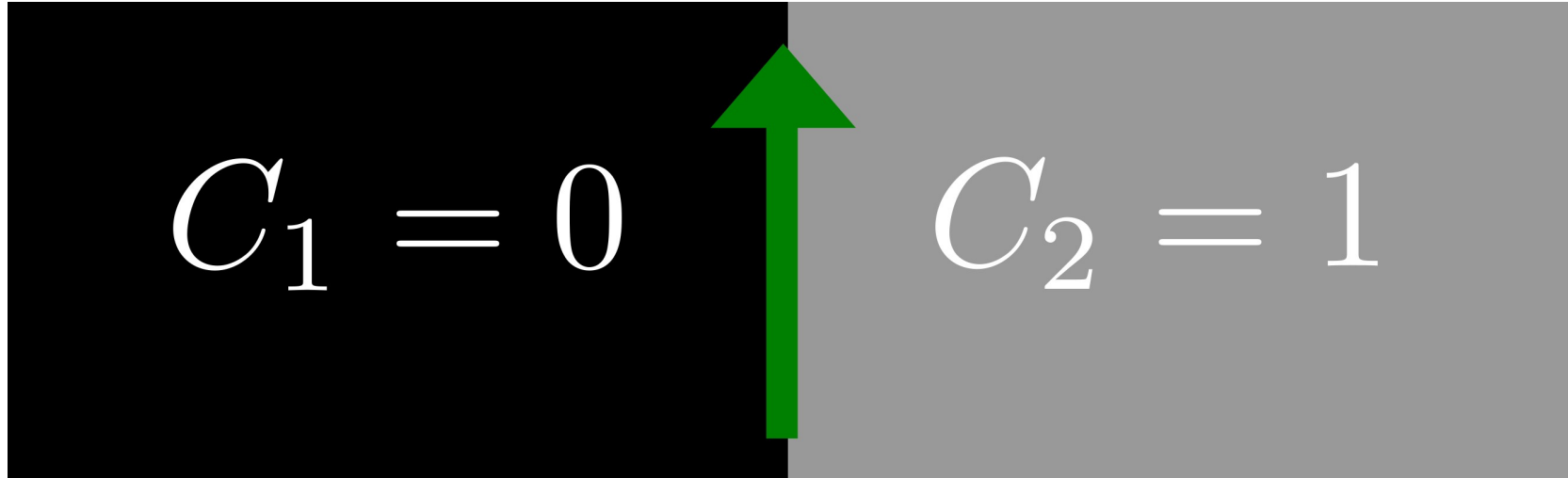


# The edge states of a Chern insulator

Trivial system

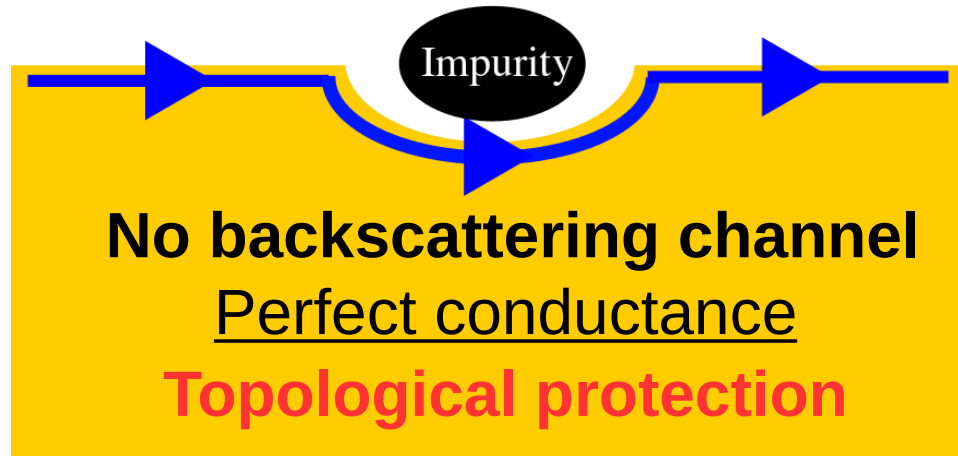
Topologically  
protected  
chiral state

Topological system



The edge states of the quantum Hall effect are topological excitations

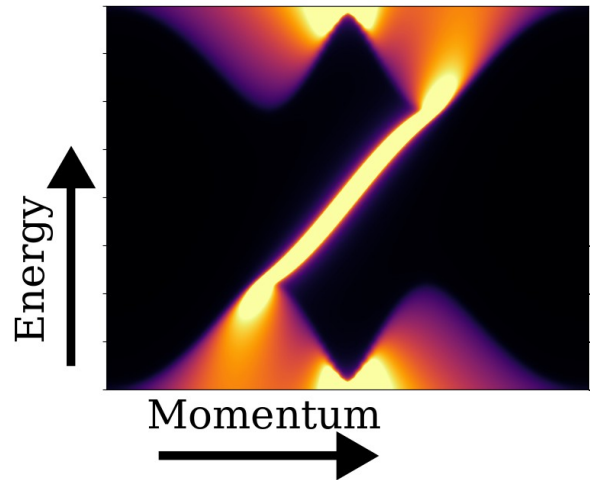
# The edge states of a Chern insulator



The edge states of the quantum Hall effect are topologically protected

# Three important topological materials

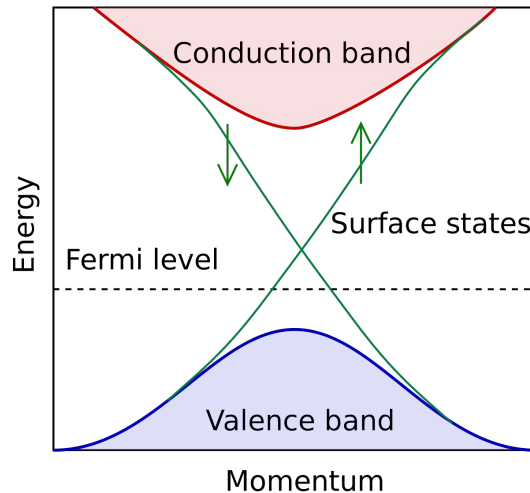
## Chern insulators



***Chiral states***

*Electronics*

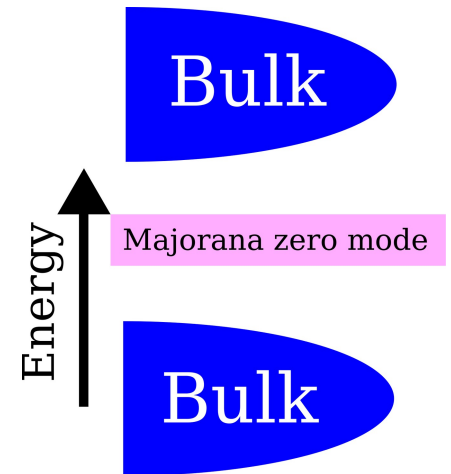
## Quantum spin Hall insulators



***Helical states***

*Spintronics*

## Topological superconductors



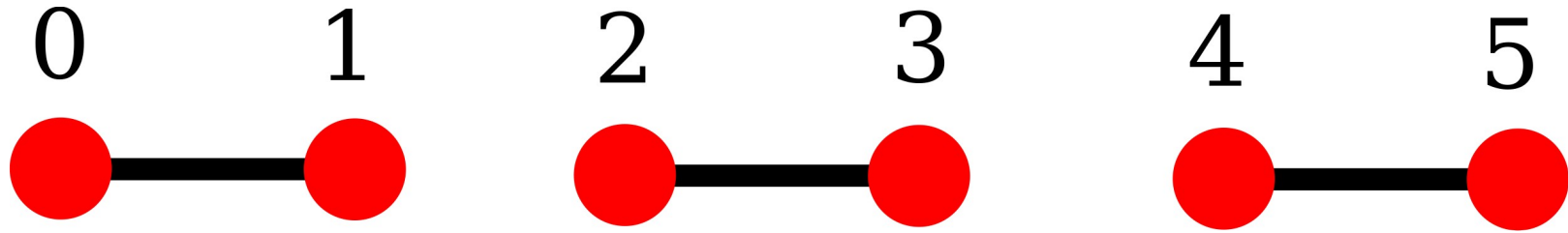
***Majorana excitations***

*Topological quantum  
computing*

# A minimal model for a topological insulator: the SSH model



# The SSH model



$$H = \sum_{n=0}^4 [1 + (-1)^n] / 2 c_n^\dagger c_{n+1} + h.c.$$

$$H = c_0^\dagger c_1 + c_2^\dagger c_3 + c_4^\dagger c_5 + h.c.$$

**Let us consider a finite dimerized chain**

# The two phases of the dimerized model

**“Trivial” phase (gaped everywhere)**

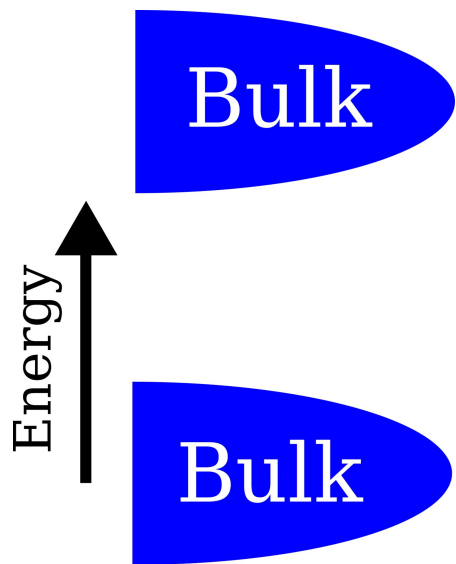


**“Topological” phase (gapless zero modes)**

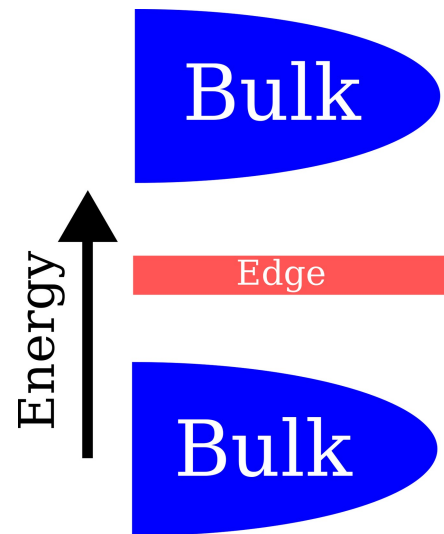


# The two phases of the dimerized model

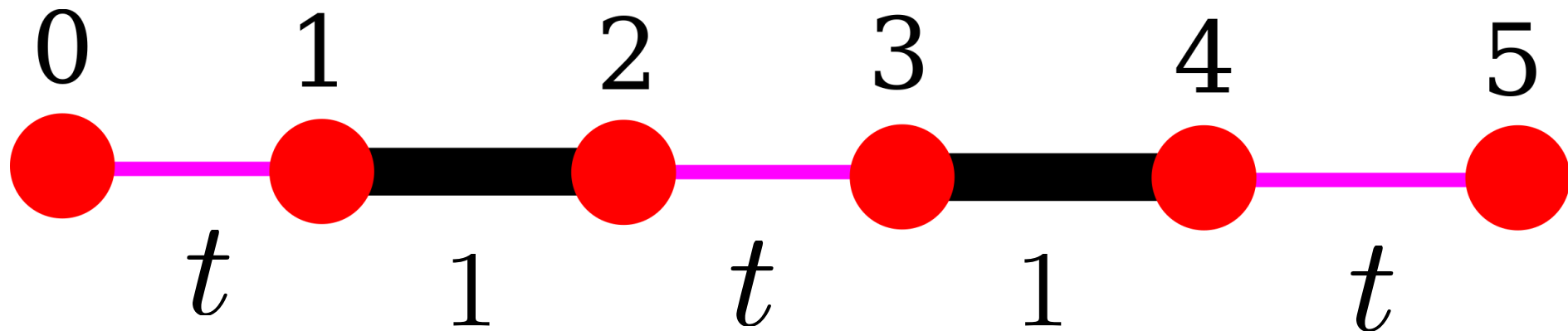
Trivial



Topological



# Coupling the dimers

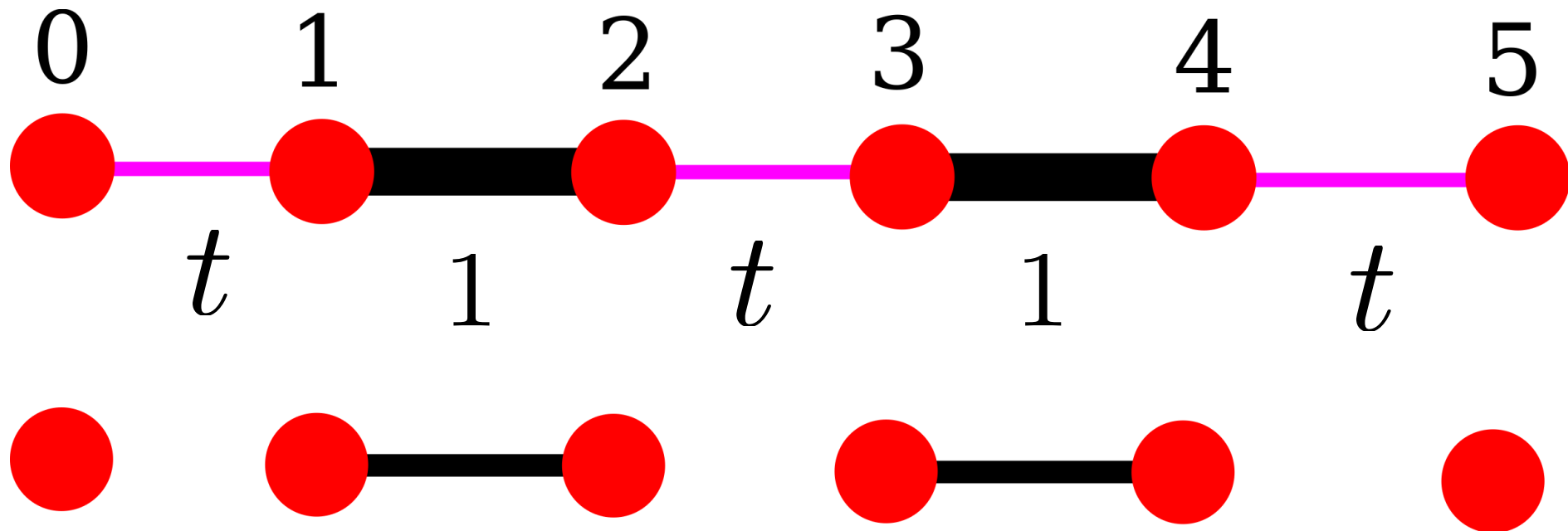


$$H = tc_0^\dagger c_1 + c_1^\dagger c_2 + tc_2^\dagger c_3 + c_3^\dagger c_4 + h.c.$$

**Does this Hamiltonian have a surface zero mode?**



# Coupling the dimers

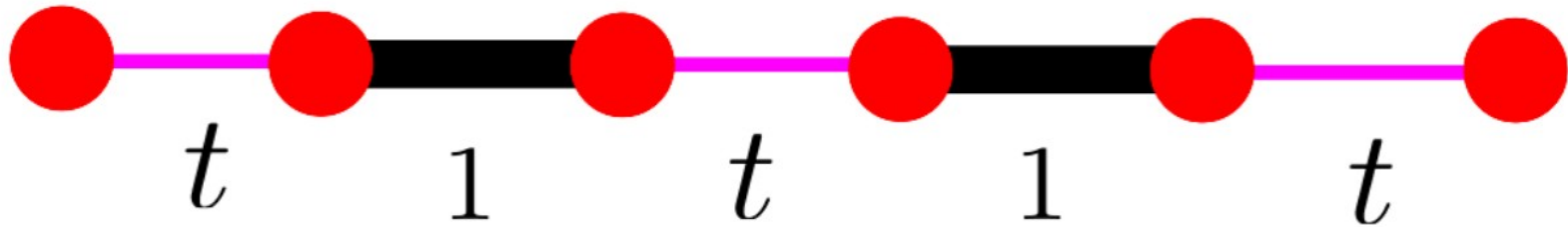


**For  $t < 1$ , both Hamiltonians are topologically equivalent**

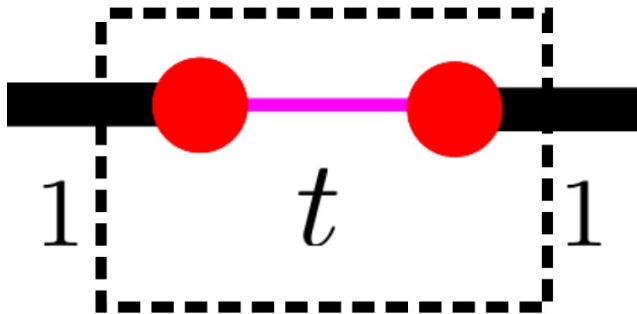
*They can be deformed into one another without closing the bulk gap*

# The bulk Hamiltonian in the dimerized model

For a finite system of this form



The unit cell is



The Hamiltonian is

$$H = \begin{pmatrix} 0 & t + e^{ik} \\ t + e^{-ik} & 0 \end{pmatrix}$$

# The bulk invariant in the dimerized model

**Hamiltonian**

$$H = \begin{pmatrix} 0 & t + e^{ik} \\ t + e^{-ik} & 0 \end{pmatrix}$$

**Zak phase**

$$\phi = \int_{BZ} A dk$$
$$A = i \langle \Psi(k) | \partial_k | \Psi(k) \rangle$$

**Two different possible values for the Zak phase**

$$\phi = 0$$

$$t > 1$$

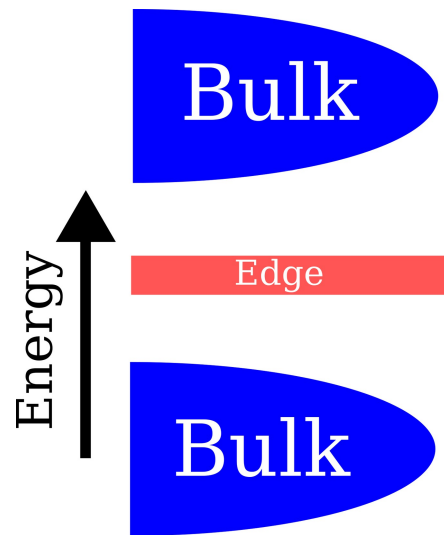
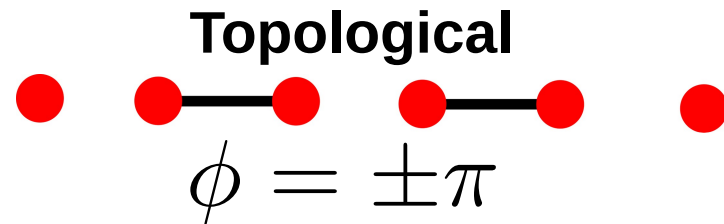
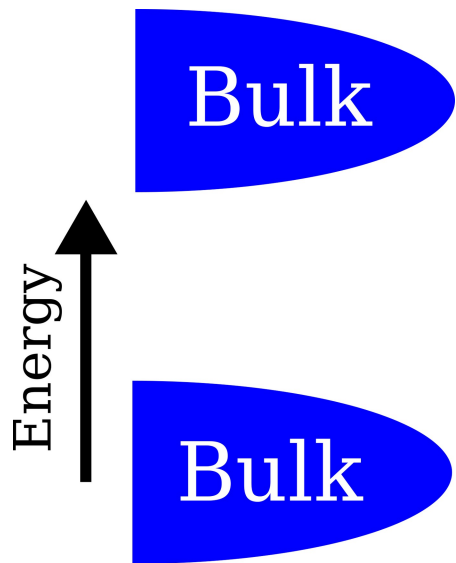
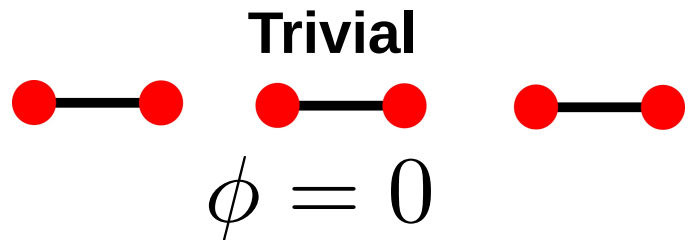
**Trivial insulator**

$$\phi = \pm\pi$$

$$t < 1$$

**Topological insulator**

# The bulk-boundary correspondence in the dimerized model



# Physical relevance of dimerized models

Some topological orders are equivalent to dimerized models upon a mathematical transformation

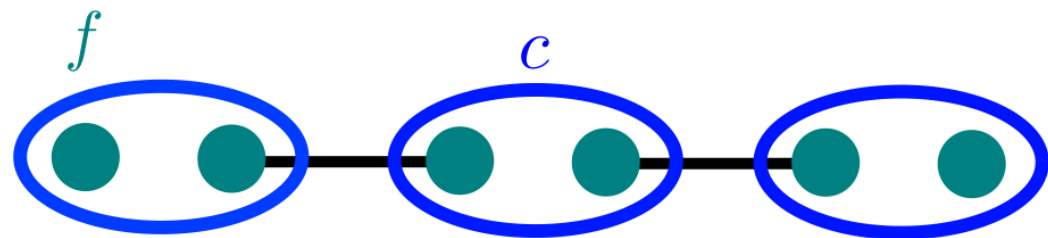
## Topological superconductor

$$H = \sum_n c_n^\dagger c_{n+1} + c_n c_{n+1} + h.c.$$

Conventional  
fermion

Majorana  
operator

$$c_n = \gamma_{2n-1} + i\gamma_{2n}$$

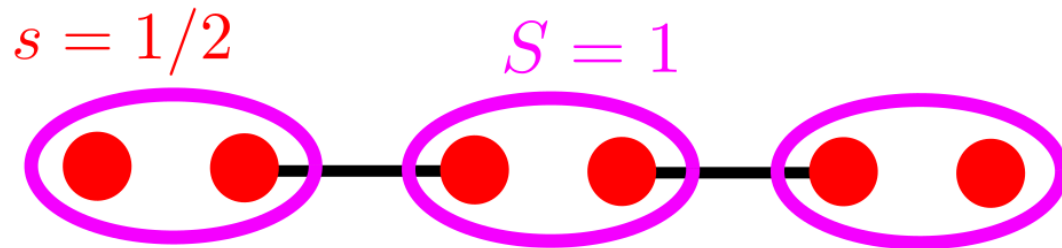


## Topological quantum magnets

$$H = \sum_n \vec{S}_n \cdot \vec{S}_{n+1}$$

$$S \sim s_1 \otimes s_2$$

$$(S=1) \quad s_{1,2} = 1/2$$





The computational challenge of  
quantum many-body systems

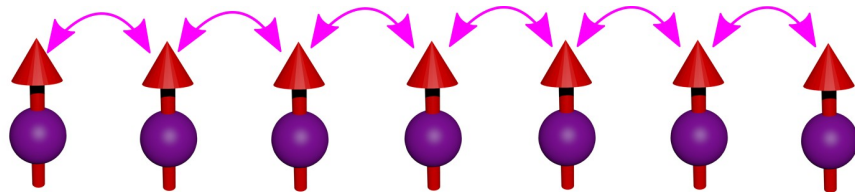
# The problem of dimensionality

In a many-body problem, the size of our vectors grows as

$$2^L$$

where  $L$  is the number of sites

$$\mathcal{H} = \sum_{\langle ij \rangle} J \vec{S}_i \cdot \vec{S}_j$$



For a single-particle tight binding problem, we can reach up to  $10^8$  sites in a laptop

For a many-problem, we cannot even store states for systems bigger than  $L = 30$  sites

$$2^{30} \sim 10^9$$

# The quantum many-body problem

Let us take a simple many-body problem

$$\mathcal{H} = \sum_{ij} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

And let us imagine that we have L different sites on our system and  $S=1/2$

For example, for  $L=2$  sites the elements of the basis are

$$|\uparrow\uparrow\rangle \quad |\uparrow\downarrow\rangle \quad |\downarrow\uparrow\rangle \quad |\downarrow\downarrow\rangle$$

For  $L=3$  sites the elements of the basis are

$$\begin{array}{cccc} |\uparrow\uparrow\uparrow\rangle & |\uparrow\uparrow\downarrow\rangle & |\uparrow\downarrow\uparrow\rangle & |\uparrow\downarrow\downarrow\rangle \\ |\downarrow\uparrow\uparrow\rangle & |\downarrow\uparrow\downarrow\rangle & |\downarrow\downarrow\uparrow\rangle & |\downarrow\downarrow\downarrow\rangle \end{array}$$

# The quantum many-body problem

Let us take a simple many-body problem

$$\mathcal{H} = \sum_{ij} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

And let us imagine that we have L different sites on our system and  $S=1/2$

For  $L=4$  sites, the elements of the basis are

$ \uparrow\uparrow\uparrow\uparrow\rangle$	$ \uparrow\uparrow\uparrow\downarrow\rangle$	$ \uparrow\uparrow\downarrow\uparrow\rangle$	$ \uparrow\uparrow\downarrow\downarrow\rangle$
$ \uparrow\downarrow\uparrow\uparrow\rangle$	$ \uparrow\downarrow\uparrow\downarrow\rangle$	$ \uparrow\downarrow\downarrow\uparrow\rangle$	$ \uparrow\downarrow\downarrow\downarrow\rangle$
$ \downarrow\uparrow\uparrow\uparrow\rangle$	$ \downarrow\uparrow\uparrow\downarrow\rangle$	$ \downarrow\uparrow\downarrow\uparrow\rangle$	$ \downarrow\uparrow\downarrow\downarrow\rangle$
$ \downarrow\downarrow\uparrow\uparrow\rangle$	$ \downarrow\downarrow\uparrow\downarrow\rangle$	$ \downarrow\downarrow\downarrow\uparrow\rangle$	$ \downarrow\downarrow\downarrow\downarrow\rangle$

# The quantum many-body problem

Let us take a simple many-body problem

$$\mathcal{H} = \sum_{ij} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

A typical wavefunction is written as

$$|\Psi\rangle = \sum c_{s_1, s_2, \dots, s_L} |s_1, s_2, \dots, s_L\rangle$$

We need to determine in total  $2^L$  coefficients

**Is there an efficient way of storing so many coefficients?**

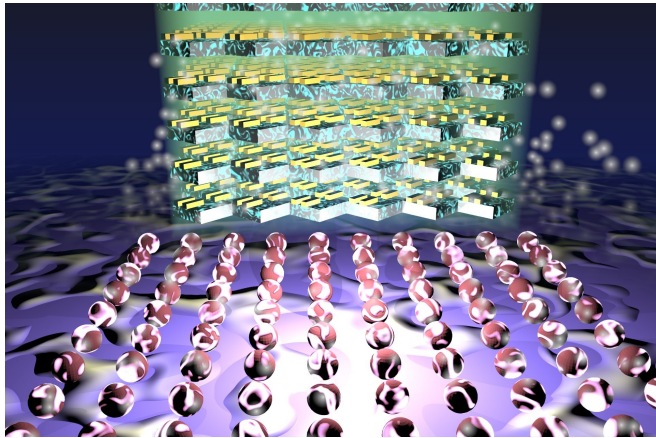


# The fundamental idea of tensor-networks

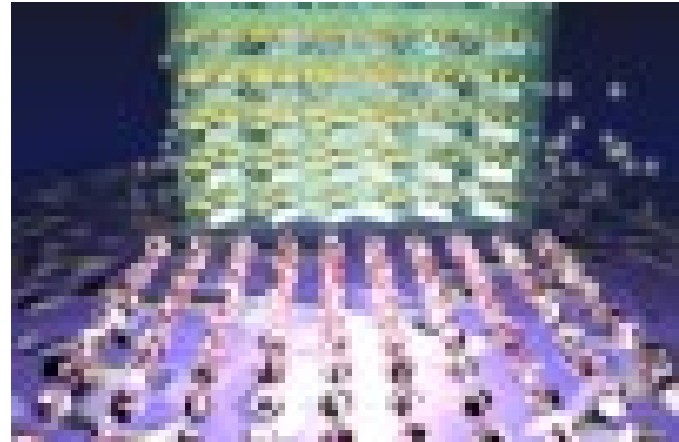
A many-body wavefunction is a very high dimensional object

$$|\Psi\rangle = \sum c_{s_1, s_2, \dots, s_L} |s_1, s_2, \dots, s_L\rangle$$

Tensor-networks allow “compressing” all that information in a very efficient way



“True wavefunction”



“Tensor-network wavefunction”

# The matrix-product state ansatz

For this wavefunction  $|\Psi\rangle = \sum c_{s_1, s_2, \dots, s_L} |s_1, s_2, \dots, s_L\rangle$

Let us imagine to propose a parametrization in this form

$$c_{s_1, s_2, \dots, s_L} = M_1^{s_1} M_2^{s_2} \dots M_L^{s_L}$$

dimension  $2^L$  dimension  $\sim Lm^2$

(m dimension of the matrix)

# State compression with tensor-networks

Given a many-body wavefunction, we can parametrize the components as

$$|\Psi\rangle = \sum_{\{s\}} \text{Tr} \left[ M_1^{(s_1)} M_2^{(s_2)} \cdots M_N^{(s_N)} \right] |s_1 s_2 \cdots s_N\rangle$$



*Matrix product state*

The previous representation allows drastically reducing the memory required to store a state

# Exponentially large algebra with tensor networks

Tensor network allow to (approximately) operate in exponentially large vector spaces

**vector**

$$|\Psi\rangle \equiv \text{[Diagram: A horizontal chain of 5 cyan squares connected by horizontal lines. Each square has a vertical line extending upwards from its top center.]}$$

**matrix**

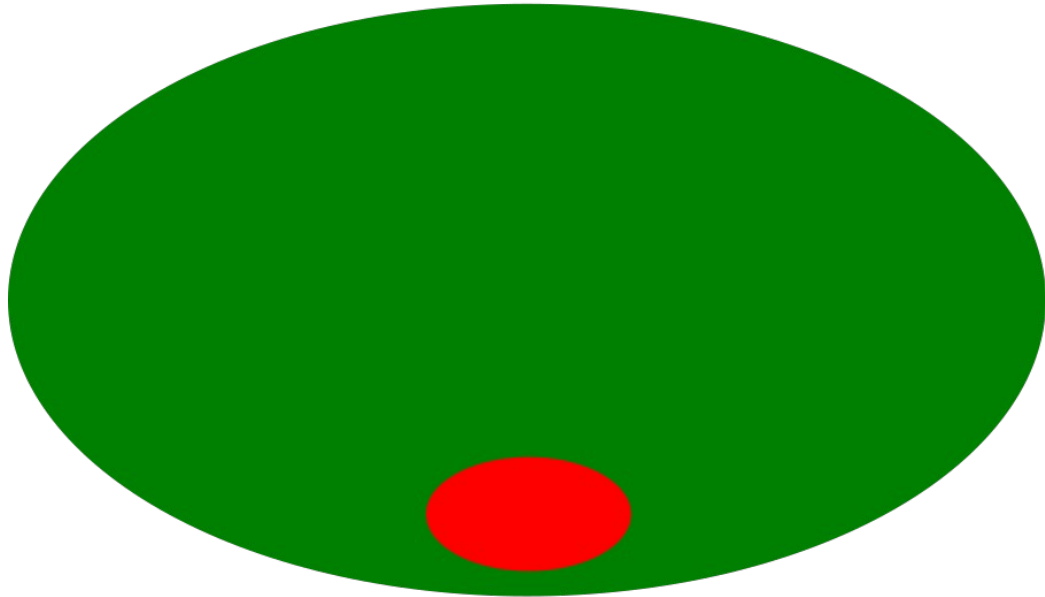
$$\hat{A} \equiv \text{[Diagram: A horizontal chain of 5 red squares connected by horizontal lines. Each square has a vertical line extending upwards from its top center and a vertical line extending downwards from its bottom center.]}$$

$$|\Psi_2\rangle = \hat{A}|\Psi_1\rangle \equiv \text{[Diagram: A 2x5 grid of squares. The top row consists of 5 red squares and the bottom row consists of 5 cyan squares. Horizontal lines connect squares in each row. Vertical lines extend upwards from the top row and downwards from the bottom row.]} = \text{[Diagram: A horizontal chain of 5 blue squares connected by horizontal lines. Each square has a vertical line extending upwards from its top center and a vertical line extending downwards from its bottom center.]}$$

$$\hat{A}_3 = \hat{A}_1 + \hat{A}_2 \equiv \text{[Diagram: A 2x5 grid of squares. The top row consists of 5 red squares and the bottom row consists of 5 orange squares. Horizontal lines connect squares in each row. Vertical lines extend upwards from the top row and downwards from the bottom row.]} + \text{[Diagram: A horizontal chain of 5 orange squares connected by horizontal lines. Each square has a vertical line extending upwards from its top center and a vertical line extending downwards from its bottom center.]} = \text{[Diagram: A horizontal chain of 5 teal squares connected by horizontal lines. Each square has a vertical line extending upwards from its top center and a vertical line extending downwards from its bottom center.]}$$

# MPS as a parametrization of area law states

Full Hilbert space



Tensor network states

$$c_{s_1, s_2, \dots, s_L} = M_1^{s_1} M_2^{s_2} \dots M_L^{s_L}$$

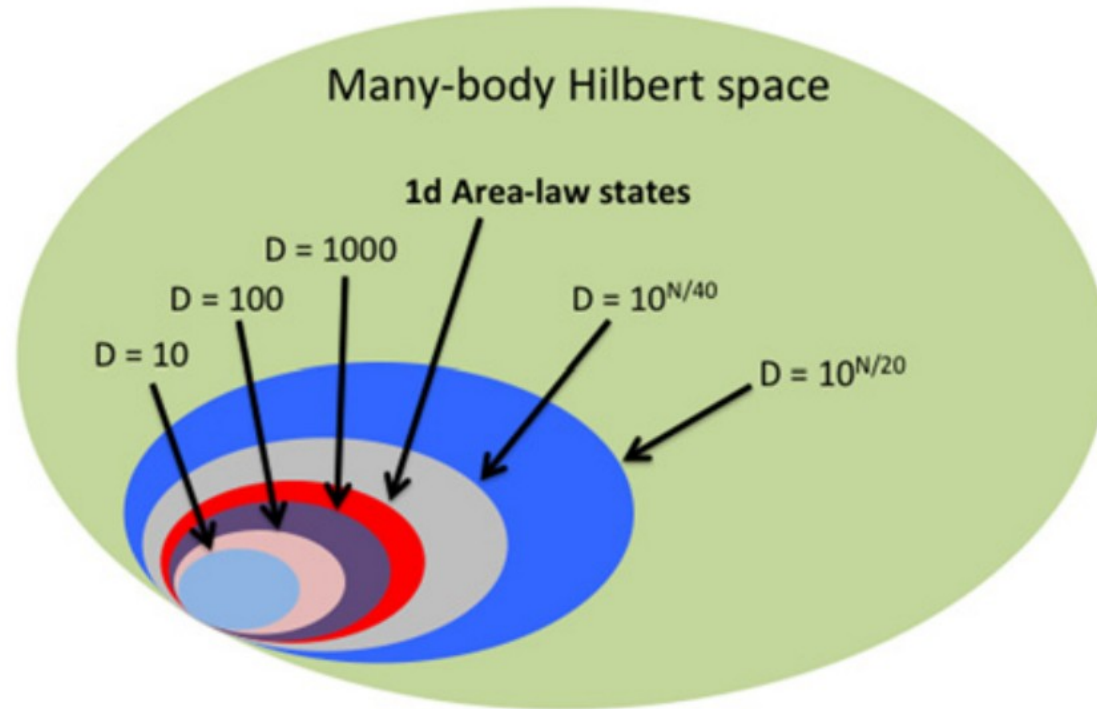
MPS have an entanglement entropy  
bounded by the bond dimension

$$S \sim \log(m)$$



# A controlled way of parametrizing the Hilbert space

Sketch of the space parametrized with bond dimension  $D$



# The matrix-product state ansatz

- This ansatz enforces a maximum amount of entanglement entropy in the state  $S \sim \log m$
- One-dimensional problems have ground states that can be captured with this ansatz

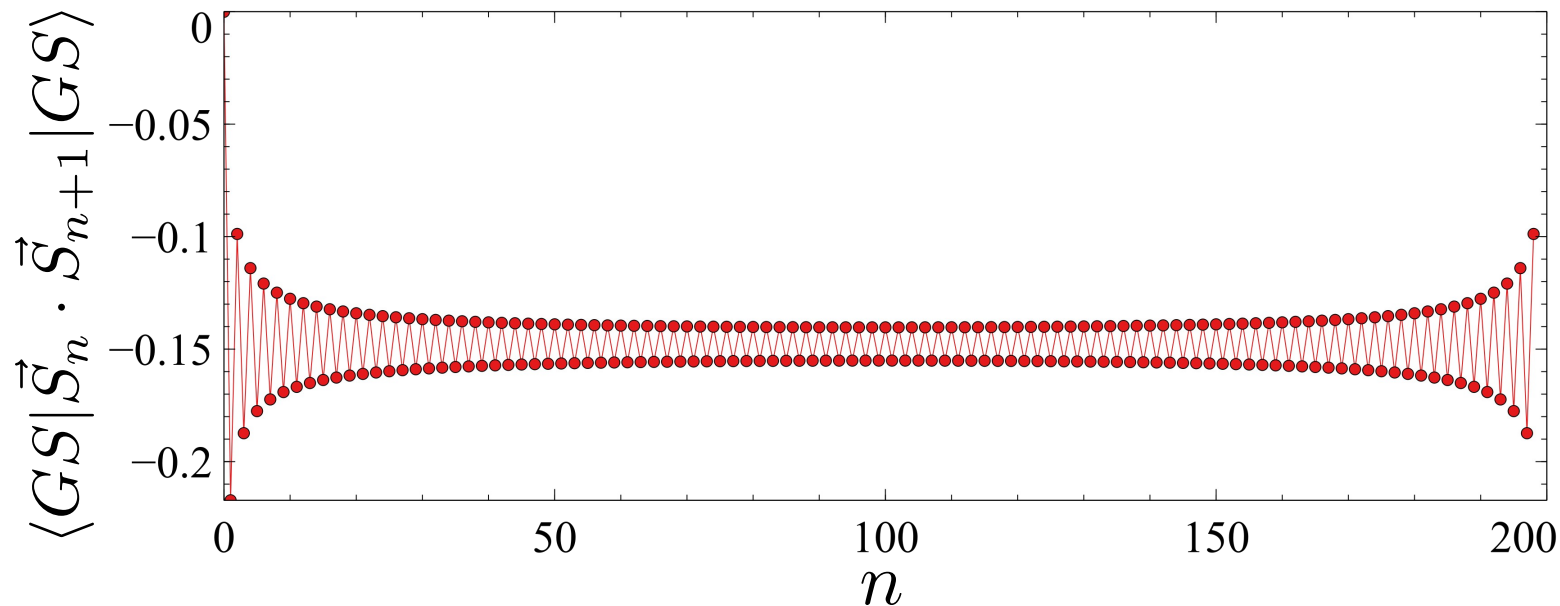
$$c_{s_1, s_2, \dots, s_L} = M_1^{s_1} M_2^{s_2} \dots M_L^{s_L}$$

This ansatz can be generalized for time-evolution, excited states, or typical thermal states

# The Heisenberg model with tensor-networks

*Non-uniform Heisenberg model*

$$\mathcal{H} = \sum_n J(n) \vec{S}_n \cdot \vec{S}_{n+1}$$
$$J(n) = J_0 + \delta \cos \Omega n$$

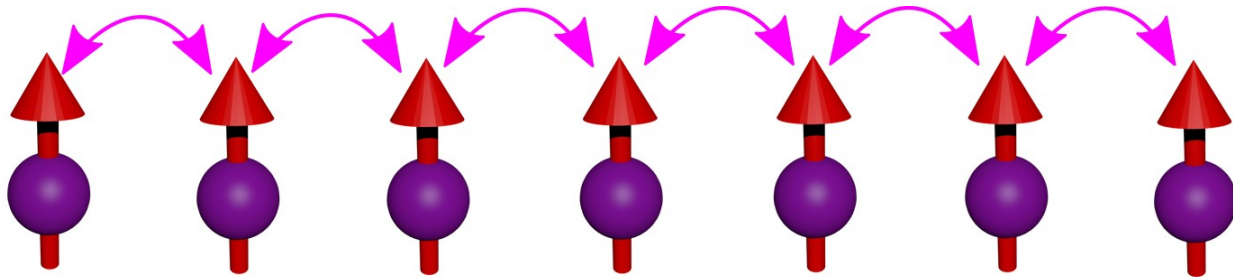


Tensor networks allow solving a 200 many-body spin model in a few seconds in a laptop

# Many-body dynamical correlators

*One dimensional Heisenberg Hamiltonian*

$$\mathcal{H} = \sum_{\langle ij \rangle} J \vec{S}_i \cdot \vec{S}_j$$

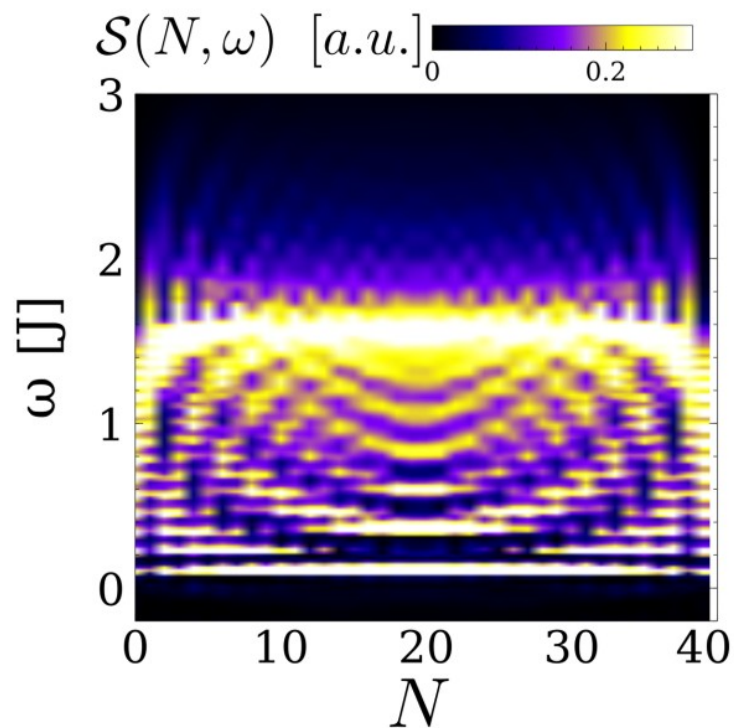


**Tensor networks allow computing dynamical correlators**

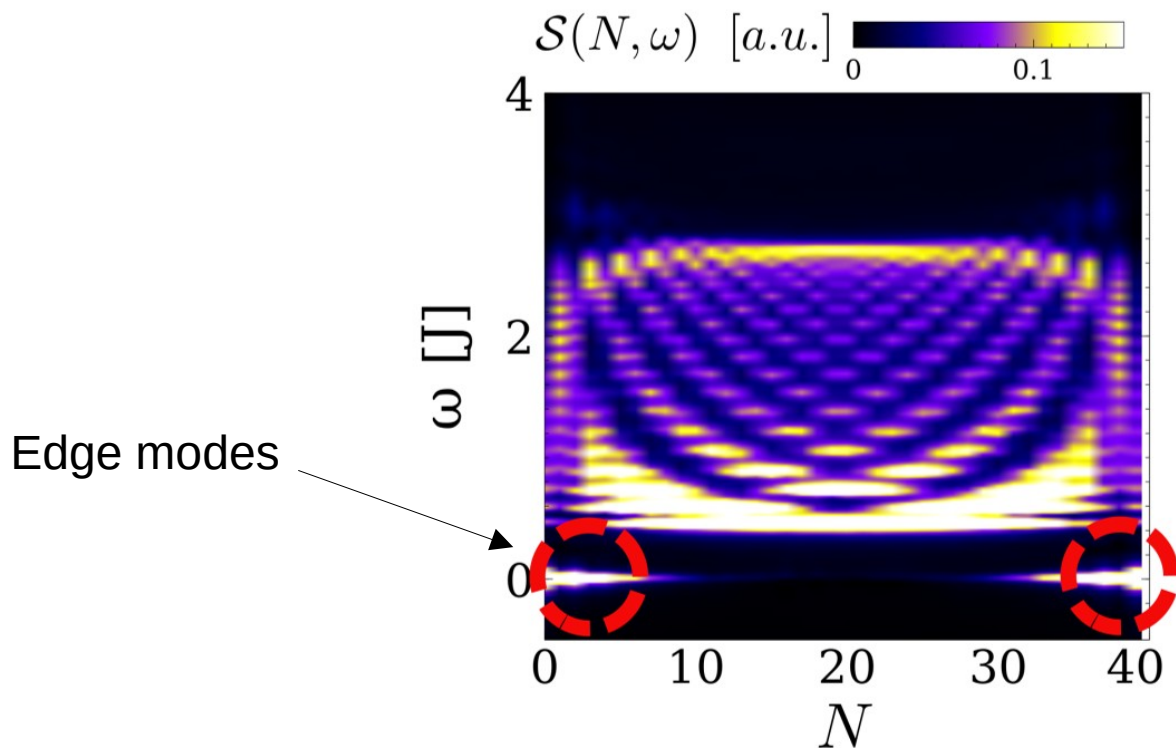
$$\mathcal{S}(N, \omega) = \langle GS | S_N^z \delta(\omega - \mathcal{H} + E_0) S_N^z | GS \rangle$$

# Dynamical structure factor of a Heisenberg model

**S=1/2 chain**



**S=1 chain**





# Open-source software for tensor-network many-body algorithms

## dmrgpy

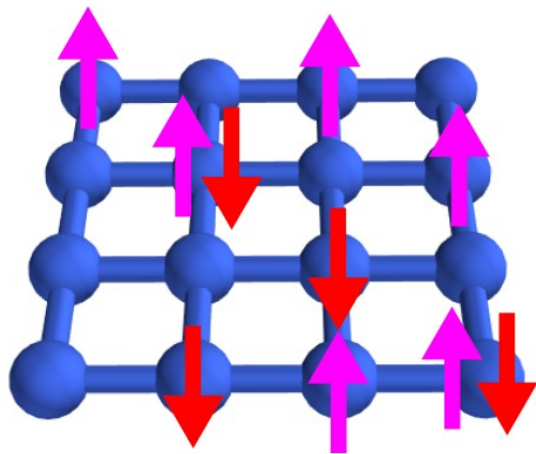
```
from dmrgpy import spinchain
spins = ["S=1" for i in range(40)] # S=1 chain
sc = spinchain.Spin_Chain(spins) # create spin chain object
h = 0 # initialize Hamiltonian
for i in range(len(spins)-1):
    h = h + sc.Sx[i]*sc.Sx[i+1]
    h = h + sc.Sy[i]*sc.Sy[i+1]
    h = h + sc.Sz[i]*sc.Sz[i+1]
sc.set_hamiltonian(h)
sc.get_dynamical_correlator(name=(sc.Sz[0], sc.Sz[0]))
```

*Generic Python library for tensor-network kernel polynomial algorithms for spins, fermions, parafermions, with static and dynamical solvers*

<https://github.com/joselado/dmrgpy>

# Some paradigmatic problems solved with matrix product states

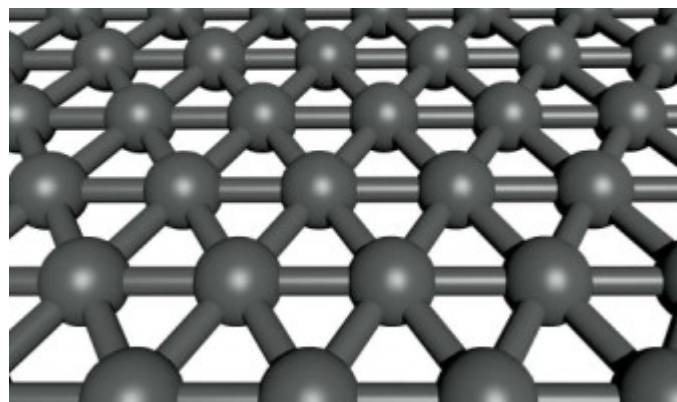
*Solving the 2D Hubbard model at finite doping*



$$H = \sum_{ij,s} t_{ij} c_{is}^\dagger c_{js} + \sum_i U c_{i\uparrow}^\dagger c_{i\uparrow} c_{i\downarrow}^\dagger c_{i\downarrow}$$

*Science*, 365(6460), 1424-1428 (2019)

*Solving the 2D Heisenberg model in frustrated lattices*

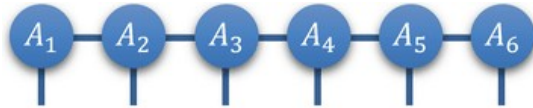


$$\mathcal{H} = \sum_{ij} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

*Phys. Rev. Lett.* 123, 207203 (2019)

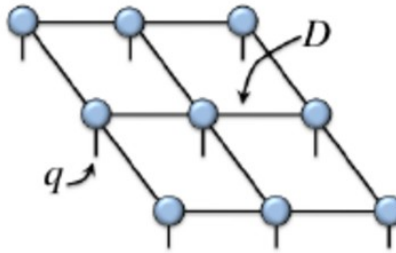
# Many-body state compression

Matrix-product states



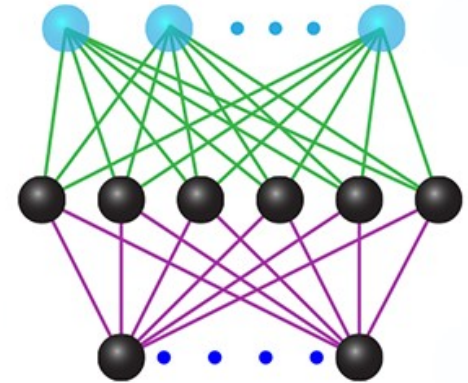
*Phys. Rev. Lett.* 69, 2863 (1992)

Projected entangled pair-states



*Annals of Physics* 326, 96 (2011)

Neural-network quantum states



*Science* 355.6325 (2017): 602-606.

***Other compressed many-body states could be potentially used for noisy quantum circuit simulation***

# Plan for today

- Emulating quantum circuits with tensor-networks

*Phys. Rev. Research 6, 033325 (2024)*

- Computing topological invariants in a superconducting quantum computer

*Phys. Rev. Research 6, 043288 (2024)*

Marcel Niedermeier



Marc Nairn



Christian Flindt



# Tensor-networks for noisy quantum circuits

# What is a quantum computer

Time evolution of quantum systems are described by  $-i\frac{\partial}{\partial t}|\Psi\rangle = H(t)|\Psi\rangle$

Whose solution is  $|\Psi(t)\rangle = e^{i\int_{t=0}^t H(t')dt'}|\Psi(t=0)\rangle$

A quantum computer is a controllable quantum system where

$|\Psi(t=0)\rangle$  and  $H(t)$  can be controlled

The results of the computation consist on observables  $\langle A \rangle = \langle \Psi(t) | A | \Psi(t) \rangle$

*Quantum computers enabling simulating dynamics that may require exponentially large resources*

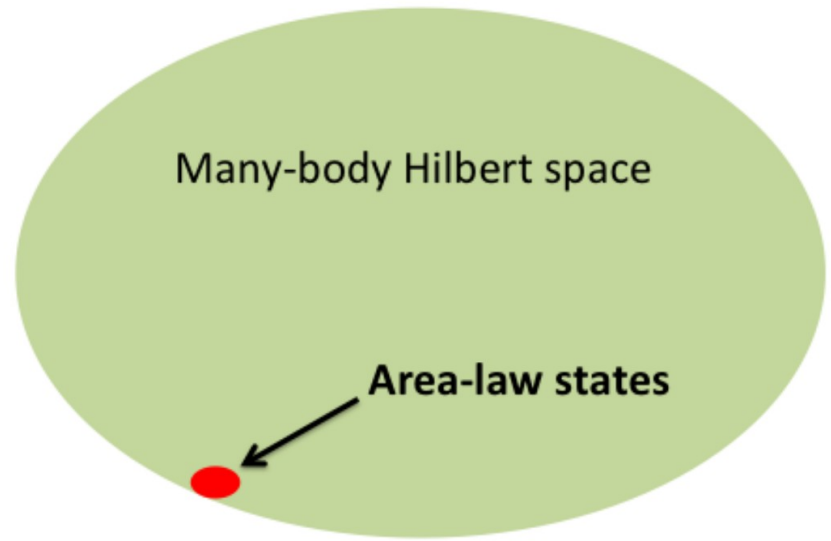
*A method to simulate large quantum many-body dynamics (tensor networks)  
also allows to simulate large quantum computers*



# How much entanglement do we need

*Qubit fidelity limits which states we can create in a quantum circuit*

*Tensor-network bond dimension limits which states we parametrize*

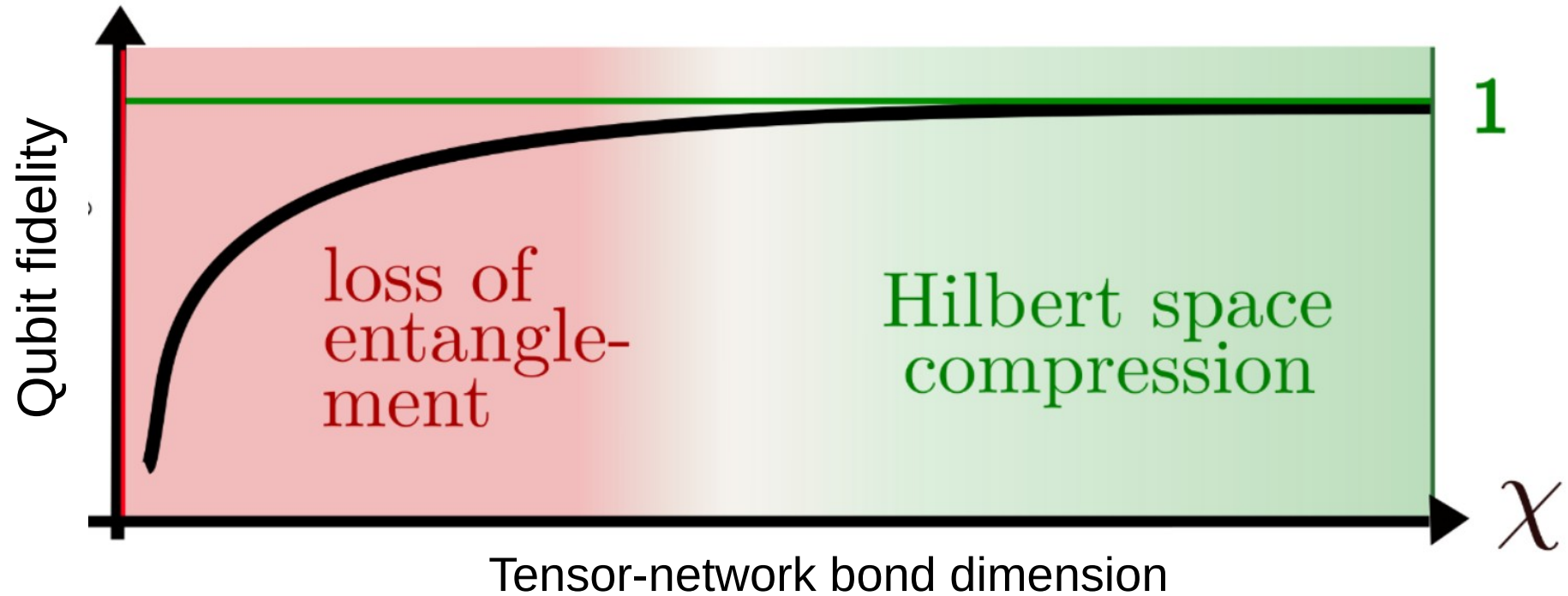


Both constraints put bounds to the entanglement of accessible states

*Phys. Rev. X 10, 041038 (2020)*

**Do we need the whole Hilbert space to successfully run a quantum algorithm?**  
**Could we get meaningful results even if we can only access part of the Hilbert space?**

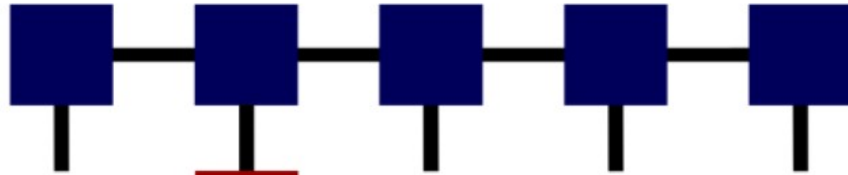
# What is the relation between a noisy qubit and a tensor network?



*The finite tensor-network bond dimension can be understood as a finite qubit fidelity*

# Applying a single qubit gate to a tensor network

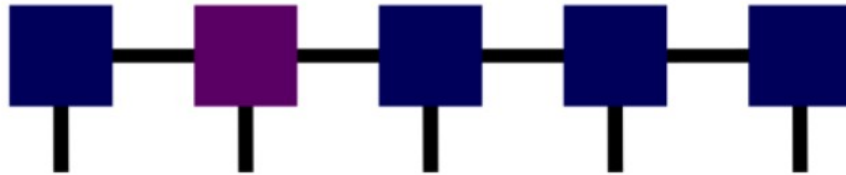
Original MPS



Single qubit gate



Transform MPS



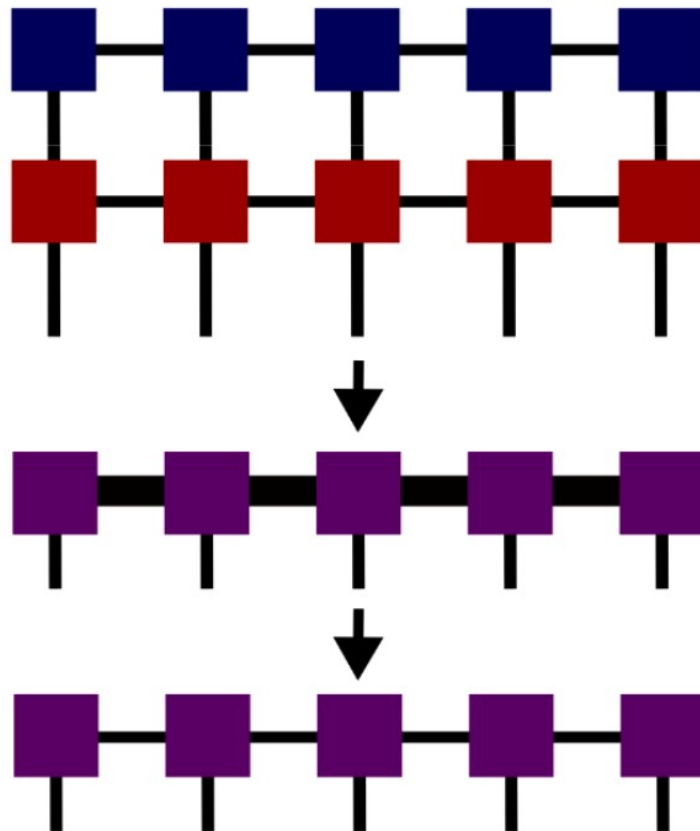
# Applying a multi-qubit gate to a tensor network

Original MPS

Multi-qubit gate

Higher dimension updated MPS

Compressed updated MPS

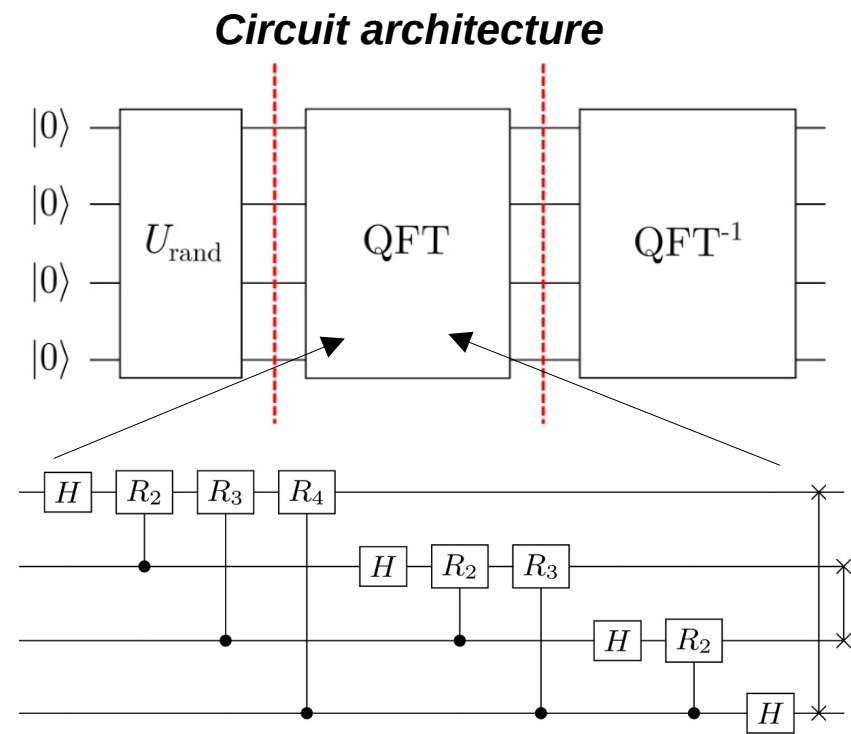
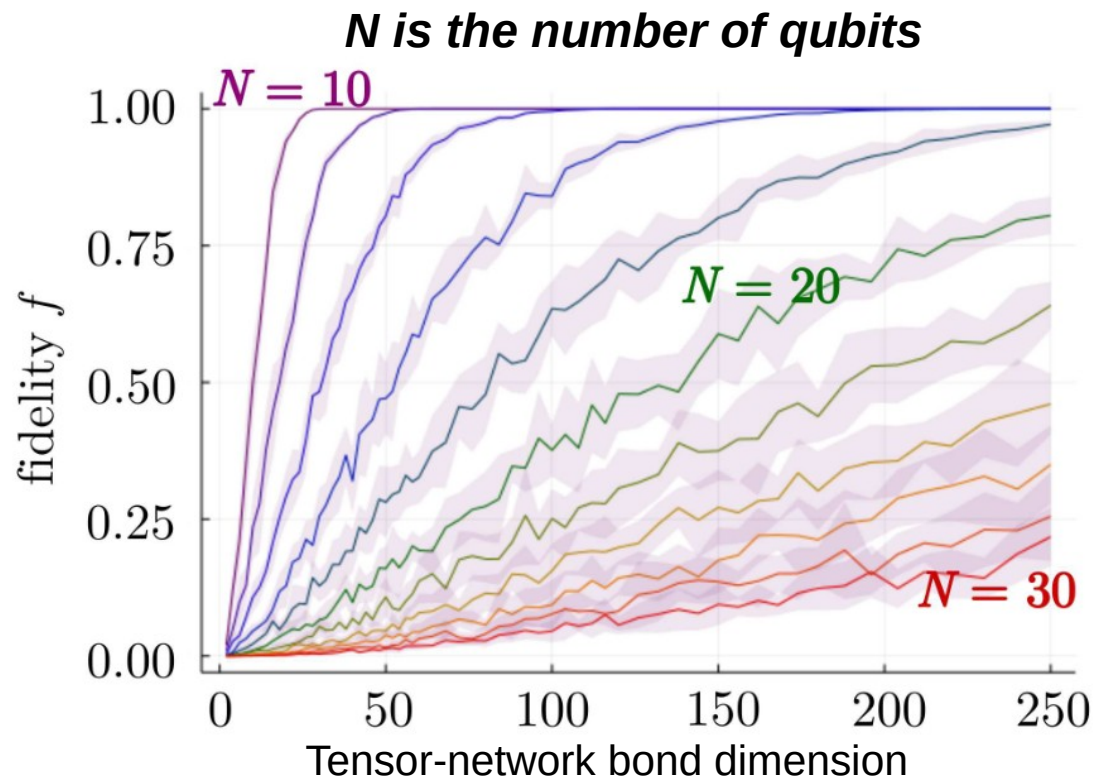


# Three quantum algorithms with tensor-network quantum circuits

- ***Quantum Fourier transform:*** quantum analogue of the discrete Fourier transform
- ***Grover's algorithm:*** find elements in a database
- ***Quantum counting algorithm:*** counting the number of solutions for a given search problem



# The quantum Fourier transform



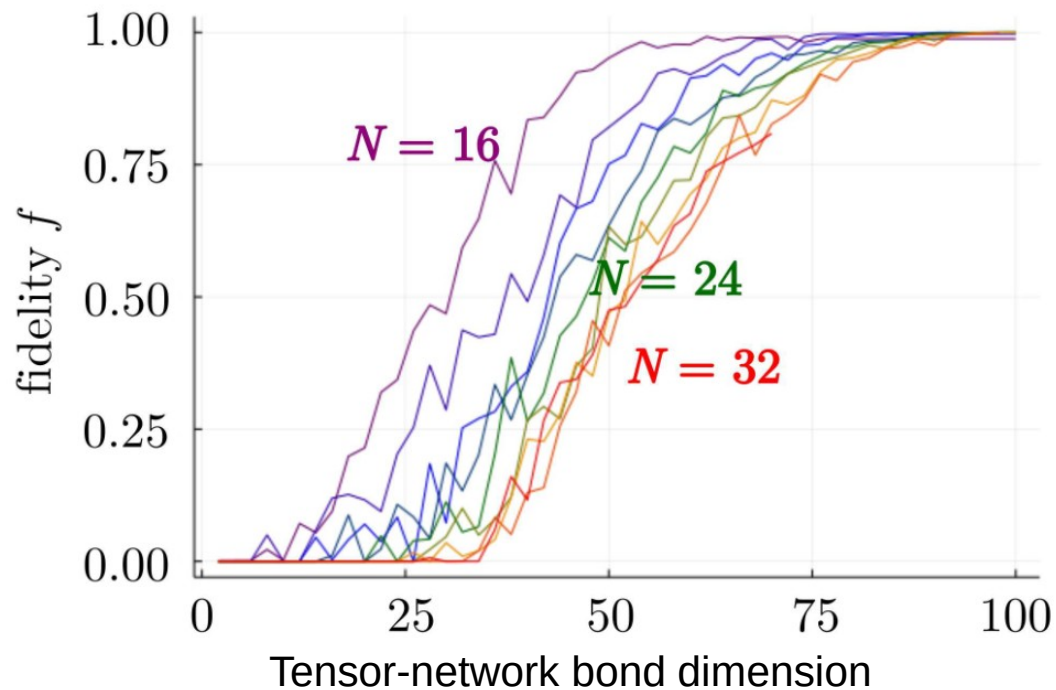
**QFT Fidelity**

$$f = |\langle \Psi'_0 | \Psi_0 \rangle|^2 \quad |\Psi'_0\rangle = \text{QFT}^{-1} \text{QFT} |\Psi_0\rangle$$



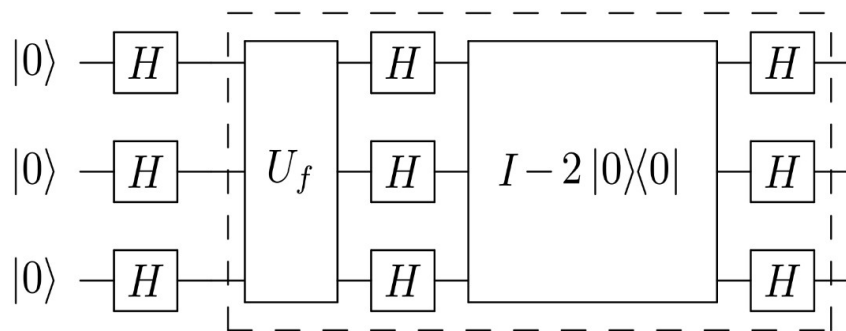
# Grover's search algorithm

***$N$  is the number of qubits***



**Circuit architecture**

Grover operator  $G$ , repeat  $r$  times



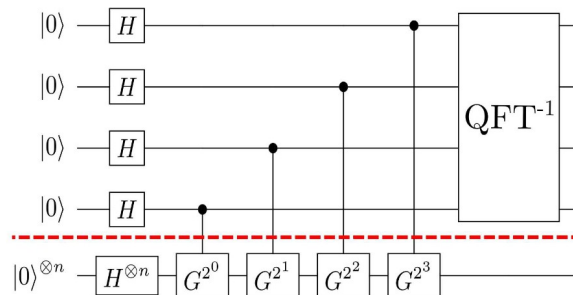
**Grover fidelity ( $m$  marked elements)**

$$f_r = \frac{1}{m} \sum_i^m |\langle \omega_i | \Psi_r \rangle|^2$$

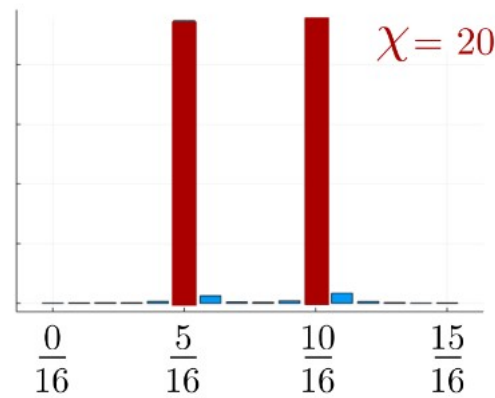
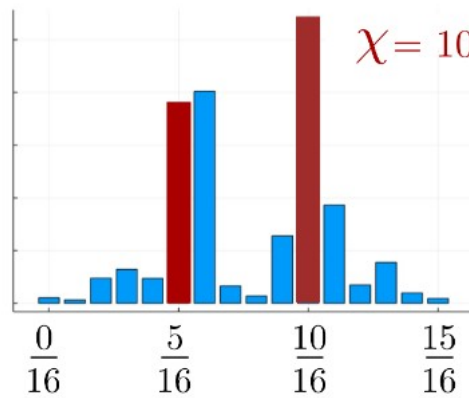
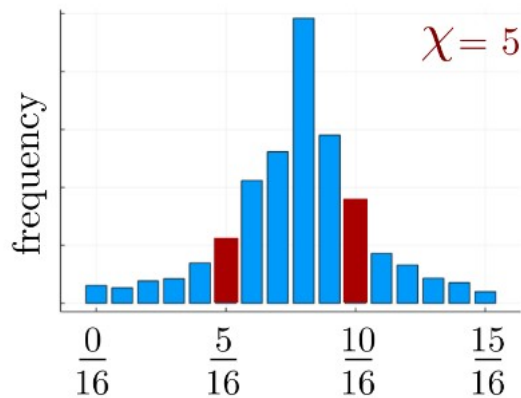
# Quantum counting algorithm

*Phys. Rev. Research 6, 033325 (2024)*

## Circuit architecture



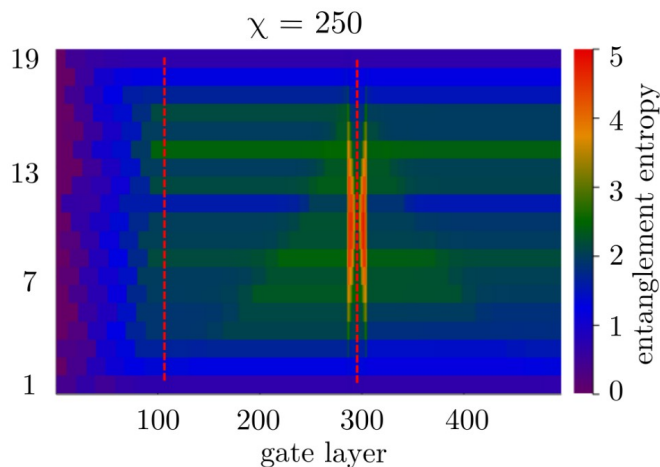
## Phases of marked elements (correct in red)



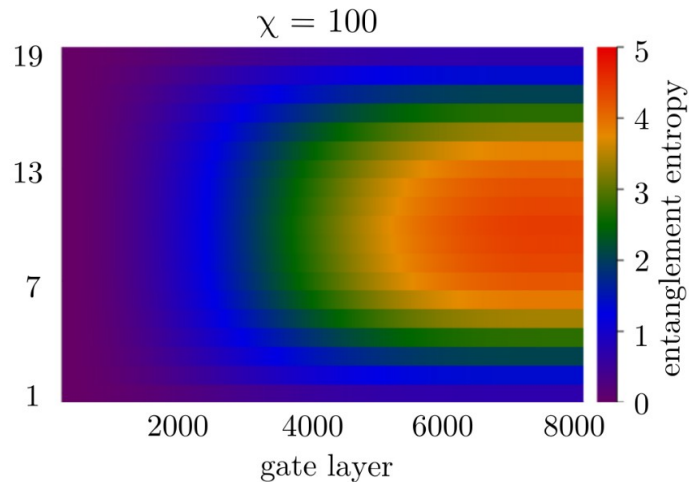
*high-quality circuit*

# Entanglement propagation in the quantum-circuit

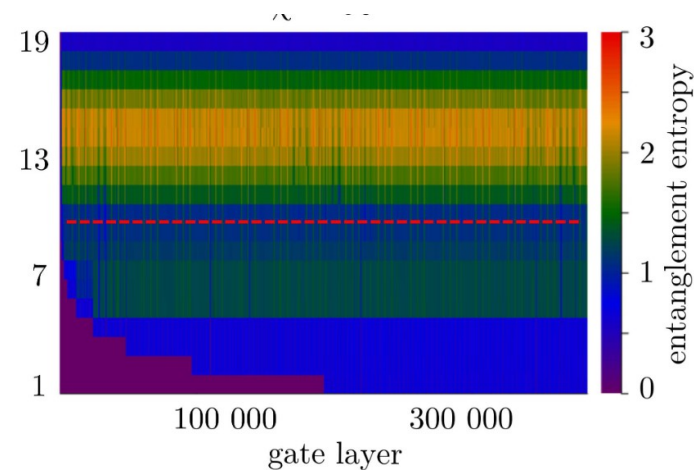
**Quantum Fourier transform**



**Grover's algorithm**



**Quantum counting algorithm**



***Entanglement builds up in concrete locations and steps of the quantum circuit***

# Quantum circuits for topological quantum materials

# Topological invariant in a Hamiltonian

We can classify Hamiltonians according to topological invariants

Hamiltonian

$$H(\mathbf{k})$$

Wavefunction

$$|\Psi(\mathbf{k})\rangle$$

$$C = \int_{BZ} \Omega d^2\mathbf{k}$$

Topological invariant  
(Chern number)

$$\Omega = \nabla \times \mathbf{A}|_z$$

Metric  
(Berry curvature)

# Berry phase with a quantum circuit

Take the eigenstates of the Hamiltonian

$$H(\mathbf{k}) |n(\mathbf{k})\rangle = E_n(\mathbf{k}) |n(\mathbf{k})\rangle$$

Evolve them in reciprocal space

$$U(T, 0) |n(\mathbf{k})\rangle = e^{i(\Theta_B - \Theta_D)} |n(\mathbf{k})\rangle$$

**Geometric phase**

$$\Theta_B = i \int_0^T dt \langle n(\mathbf{k}(t)) | \partial_t | n(\mathbf{k}(t)) \rangle$$

**Dynamical phase**

$$\Theta_D = \int_0^T dt E_n(\mathbf{k}(t))$$

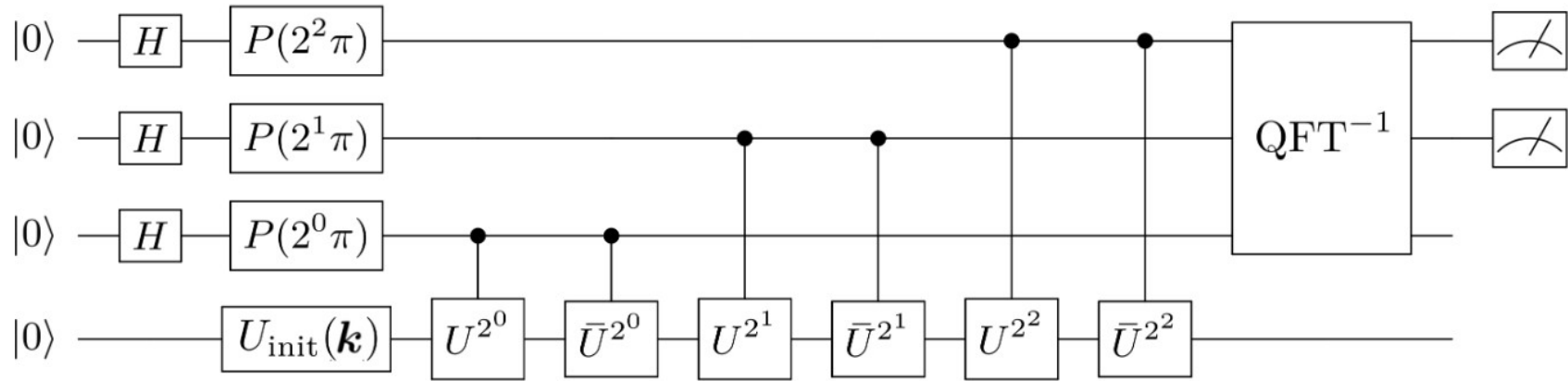
We want to cancel out the dynamical phase, so we can evolve first forward and then backwards in time

$$\bar{U}(T, 0) U(T, 0) |n(\mathbf{k})\rangle = e^{i2\Theta_B} |n(\mathbf{k})\rangle$$

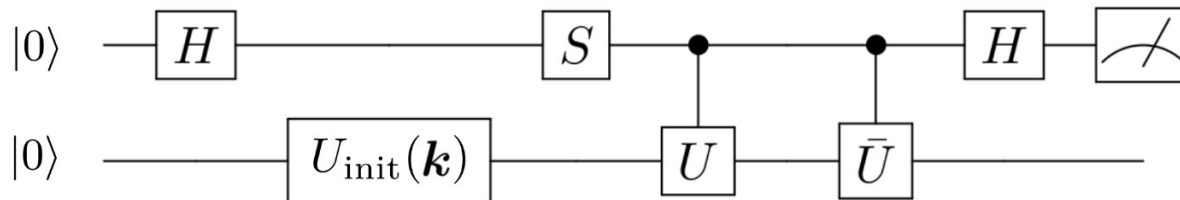


# Quantum circuits to compute topological invariants

## Berry phase with quantum phase estimation (based on QFT)



## Berry flux (based on Hadamard test)



# Two ways of computing a Chern number

**Method 1:** Integrate the Berry Curvature

$$C = \frac{1}{2\pi} \int_{BZ} d^2\mathbf{k} \Omega(\mathbf{k})$$

**Method 2:** Pumping of the Wannier charge centers

$$X_W(k_y) = i \int_{-\pi}^{\pi} dk_x \langle n(\mathbf{k}) | \partial_{k_x} | n(\mathbf{k}) \rangle / 2\pi$$

(and count how many times they wind)

# Five implementations of the topological invariant

- Helmi quantum computer
- Exact quantum circuit emulator
- Noisy quantum circuit emulator
- Tensor-network quantum circuit emulator
- Exact classical algorithm

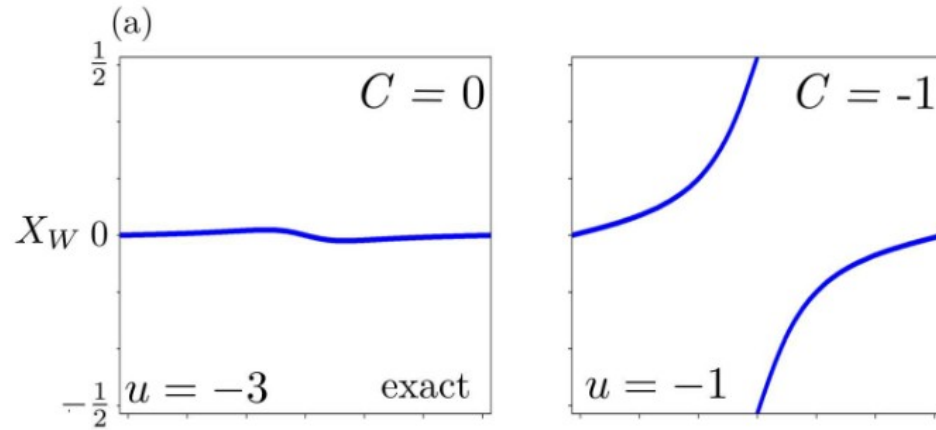
# Helmi quantum computer



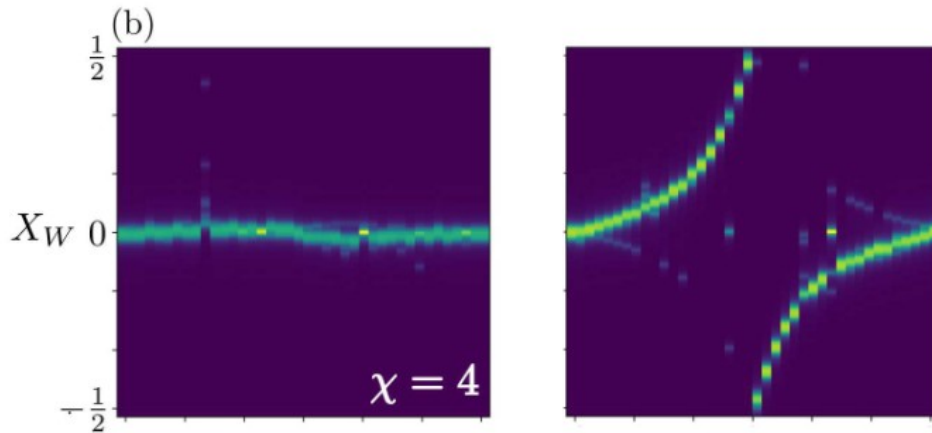
*Helmi, the first Finnish quantum computer, co-developed by VTT and IQM Quantum Computers, is operated by VTT in Espoo, Finland. Helmi is based on superconducting technology, and provides five qubits.*

<https://fiqci.fi/>

# Chern number from Wannier winding

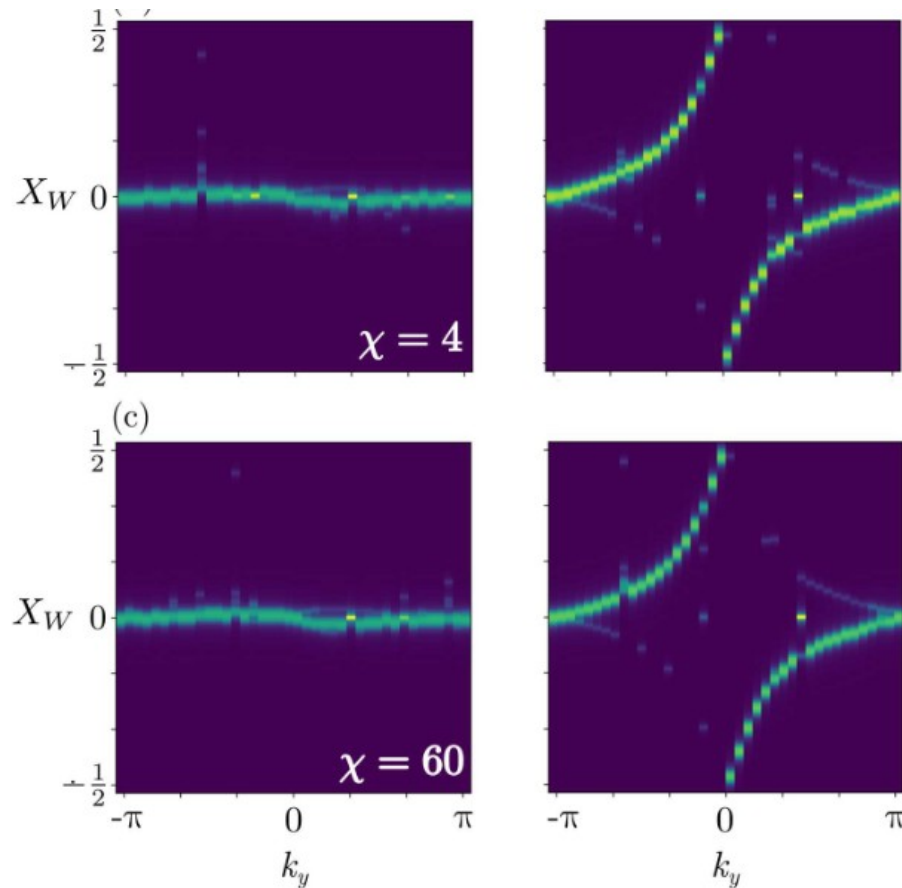


**Exact classical simulation**



**Tensor network simulator**

# Chern number from Wannier winding



**Low bond dimension**

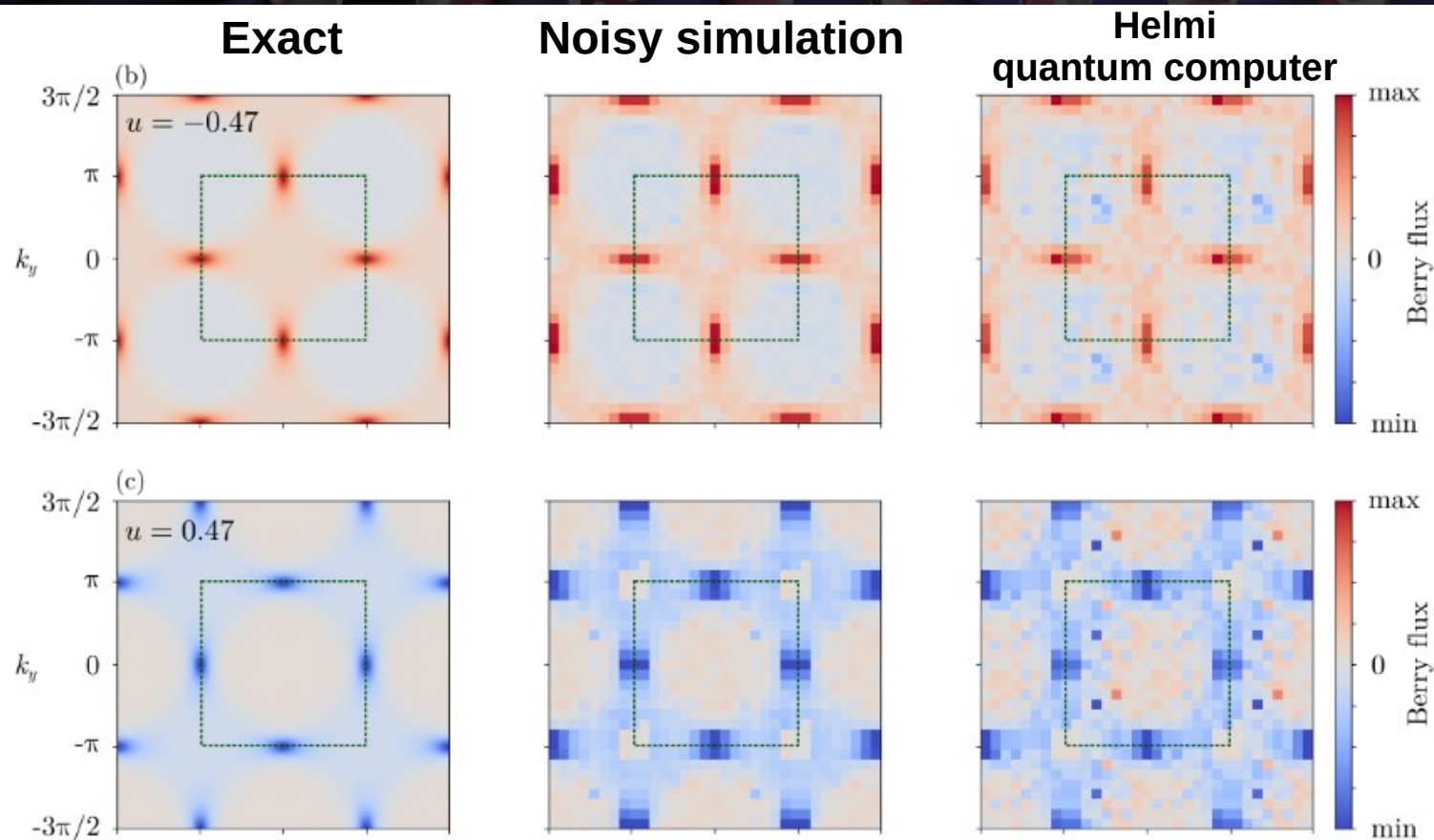
*Tiny part of the Hilbert space*

**High bond dimension**

*Most of the Hilbert space*



# Chern number from Berry curvature integration

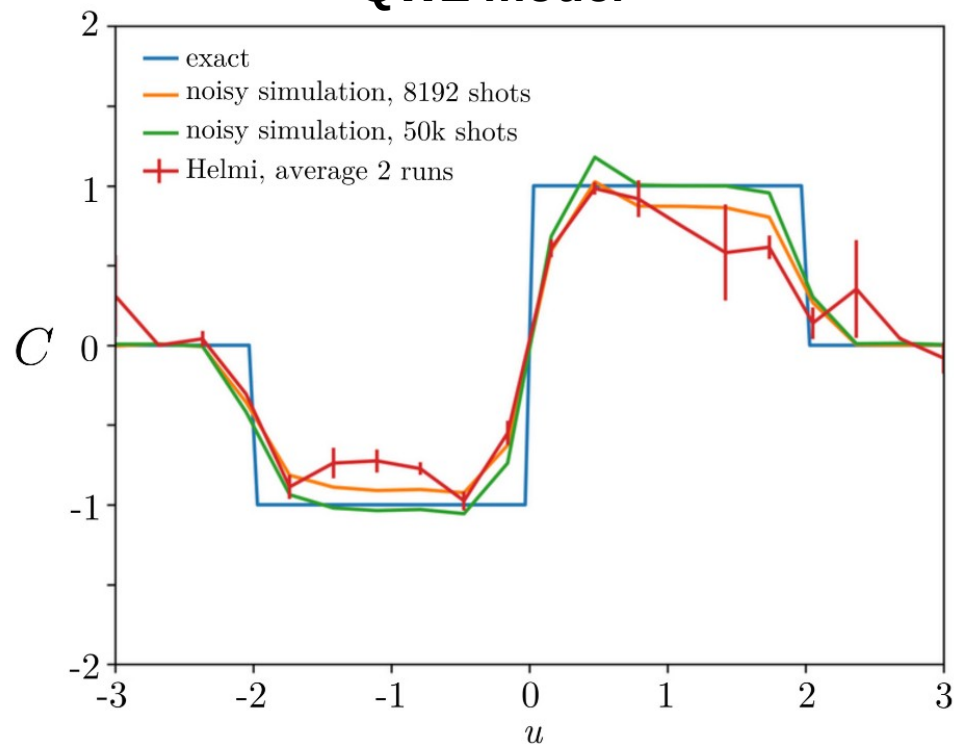


$$C = 1$$

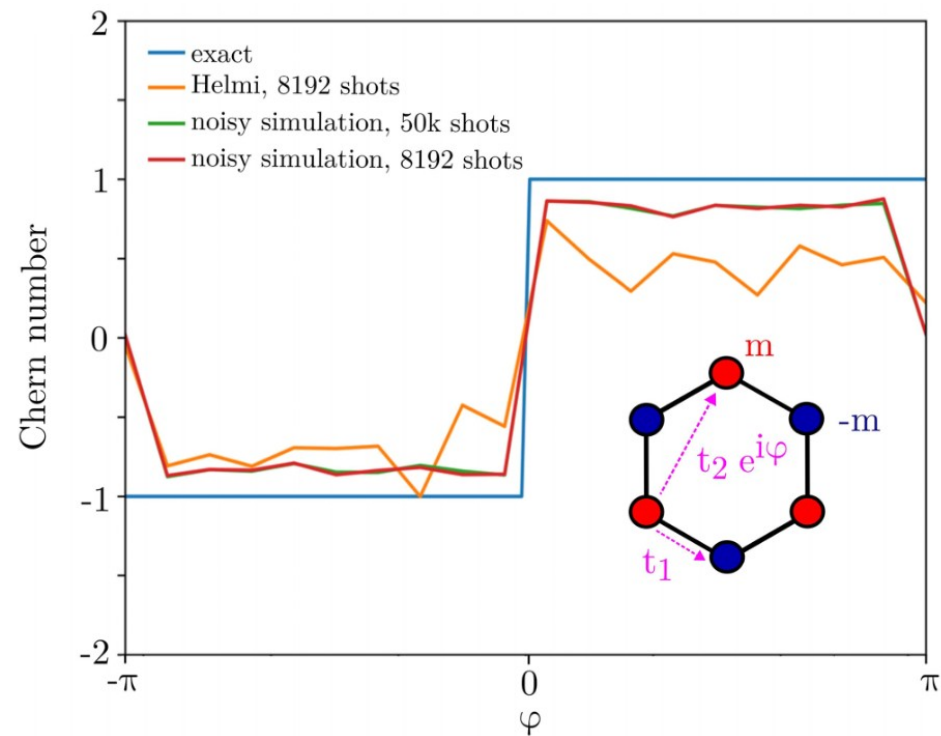
$$C = -1$$

# Topological phase diagram

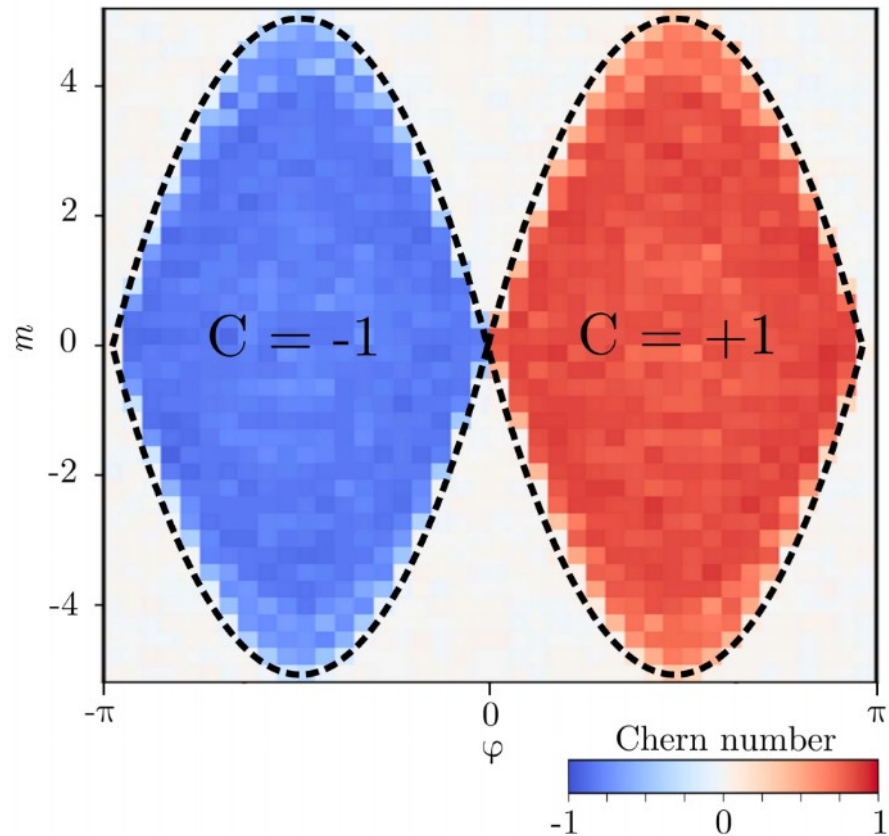
## QWZ model



## Haldane model



# Topological phase diagram

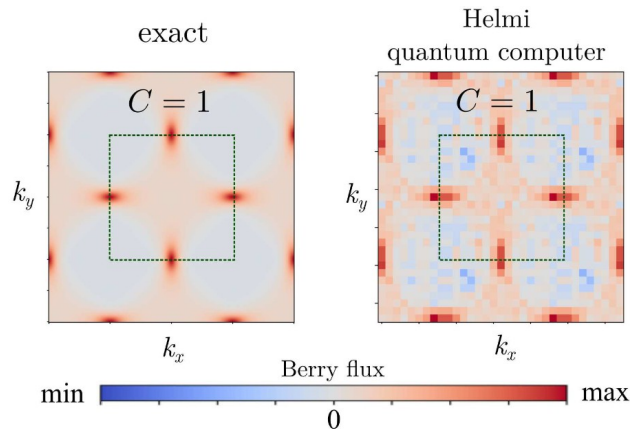
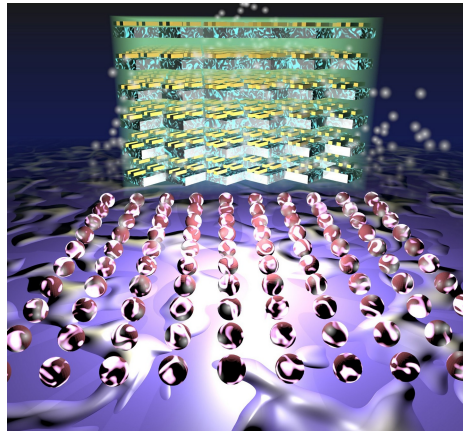


## Haldane model

Noisy calculations provide qualitatively correct topological phase diagrams

# Take home

Currently available quantum computers allow computing topological invariants, and can be effectively benchmarked with tensor-networks



Phys. Rev. Research 6, 033325 (2024)  
Phys. Rev. Research 6, 043288 (2024)

**Funding from**

Finnish Quantum Flagship

