Qubit array embedded in a cavity

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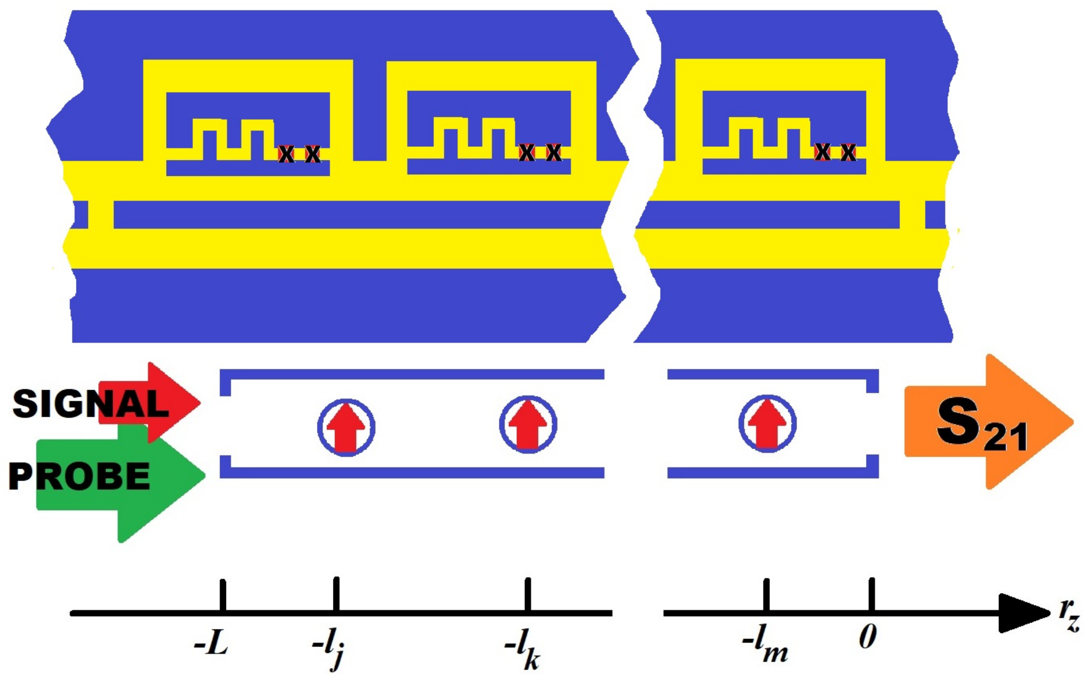
EU projects: "QRC"

Motivations

- Precise detection and characterization of weakly intense photons beams:
  $\rightarrow$ Enhanced amplification with many qubits
- Detector description starting from fundamental QED:
  Cooper pair field, photon field $\neq$ phenomenology
- Project SUPERGALAX: Detection of axions:
  elementary particle predicted to avoid strong CP violation in QCD
  Assumption: axion + magnetic field $\rightarrow$ microwave photon
- Project QRC: in-out transformation for reservoir computing
- Non linearity: interaction between MW photons
- Modelling qubits measurements using QND (readout)

Setup: waveguide in a array of transmon qubits



General use: qubit measurement or single photon detection

A weak signal: resonant photon beams interact with qubits in cavity of size L

Quantum non demolition (QND) measurement:

Readout of Stark shift of qubit by measuring the transmission S_{21}
$\rightarrow$ using off-resonant probe beam

General theory

$$\partial_t\hat{\rho}(t) + i[\hat{H}_S(t),\hat{\rho}(t)]$$
$$= -\frac{\gamma_c}{2}\left[\{\hat{a}^\dagger\hat{a},\hat{\rho}(t)\} - 2\hat{a}\hat{\rho}(t)\hat{a}^\dagger\right] - \sum_{j=1}^{N}(\frac{\Gamma_j}{2} + \Gamma_{\phi,j})\{\hat{\sigma}_j^+\hat{\sigma}_j^-,\hat{\rho}(t)\} - \Gamma_j\hat{\sigma}_j^-\hat{\rho}(t)\hat{\sigma}_j^+ . \tag{1}$$

Hamiltonian written in the rotating wave approximation (RWA) is:

$$\hat{H}_S(t) = -\omega_{0c}\hat{a}^\dagger\hat{a} + i\sum_{\pm}\sqrt{\frac{\gamma_c}{2}}(\hat{x}_{\pm}^{in*}(t)\hat{a} - x_{\pm}^{in}(t)\hat{a}^\dagger) - \sum_{j=1}^{N}\left[\frac{\Delta_j}{2}\hat{\sigma}_j^z + g_j(\hat{a}^\dagger\hat{\sigma}_j^- + \hat{a}\hat{\sigma}_j^+)\right] , \tag{2}$$

Non trivial commutation relations for black box quantum variables :

$$[\hat{x}_{\pm}^{in}(t),\hat{x}_{\pm}^{in\dagger}(t')] = \delta(t-t') \qquad [\hat{x}_{\pm}^{out}(t),\hat{x}_{\pm}^{out\dagger}(t')] = \delta(t-t') \tag{3}$$

In-out relation:

$$\hat{x}_{\pm}^{out}(t) = \sqrt{\frac{\gamma_c}{2}}\hat{a}(t) + \hat{x}_{\mp}^{in}(t) \tag{4}$$

Reservoir computing could benefit from the gigantic Fock space of the algebra of $\hat{x}_{\pm}^{in}(t)$

In-out relation: basic assumptions

- No qubit relaxation Γ_j in other channels than TEM. No given estimation!
- No dephasing $\Gamma_{\phi,j} \sim 10KHz$
- Classical approximation: coherent state of radiation
  $\widehat{a} \rightarrow \beta(t) + \delta\widehat{a}$ valid for high photon number
- Dynamics of frequency scale (MHz) much slower than any detuning ($\sim 100MHz$)

Already known result in literature for one qubit: shift frequency vs. power

$$\omega = \omega_c - \frac{g_1^2/\Delta_1}{\sqrt{1 + 4g_1^2 n_{ph}/\Delta_1^2}} \tag{5}$$

PRL 105, 100505 (2010); PRL 105, 173601 (2010); arXiv: 2310.08388; PRX 5, 031028 (2015); PRX Quantum 5, 010327 (2024)

Pocket equation: classical radiation

$V^{in}(t) = \sqrt{\hbar\omega_0 Z/2}(x^{in}(t)e^{-i\omega_0 t} + x^{in*}(t)e^{i\omega_0 t})$ Z line impedance

$V^{out}(t) = \sqrt{\hbar\omega_0 Z/2}(x^{out}(t)e^{-i\omega_0 t} + x^{out*}(t)e^{i\omega_0 t})$ ω_0 carrier frequency

$$\left[i\partial_t + \omega_0 - \omega_c - \sum_{j=1}^{N} \frac{g_j^2/\Delta_j}{\sqrt{1 + 8g_j^2|x^{out}(t)|^2/(\gamma_c \Delta_j^2)}} + i\frac{\gamma_l}{2} \right] x^{out}(t) = \frac{\gamma_c}{2i} x^{in}(t)$$

ω_c bare cavity frequency N qubit number ($1 \le j \le N$)

$\Delta_j = \omega_0 - \omega_j$ detuning of the qubit frequency j

g_j coupling of the qubit j with the cavity field

$\gamma_c = \omega_c/Q_c$ cavity leakage

$Q_c = |Q_c|e^{-i\phi}$ coupling quality factor with the mismatch phase factor ϕ.

$\gamma_l = \omega_c/Q_l$ cavity and internal leakage

$Q_l = 1/(1/Q_i + 1/|Q_c|)$ loaded quality factor with internal quality factor

Transmission $S_{21}(t) = x_{out}(t)/x_{in}(t)$: low broadening

Power $P = \hbar\omega_0 |x_{in}(t)|^2$, amplitude $x_{out}(t) = \sqrt{\gamma_c/2}\beta(t)$ and photon number $n_{ph}(t) = |\beta(t)|^2$

Steady state solution: $x^{in}(t) = x_0^{in}$

$$\beta_0 = -\frac{i x_0^{in}\sqrt{\gamma_c/2}}{\omega_s + i\gamma_l/2} \qquad x_0^{out} = \frac{i\gamma_c x_0^{in}/2}{\omega_s + i\gamma_l/2} \tag{6}$$

where

$$\omega_s = \omega_0 - \omega_c - \sum_{j=1}^{N} \frac{g_j^2/\Delta_j}{\sqrt{1 + 4g_j^2 n_{ph}/\Delta_j^2}} \tag{7}$$

$\rightarrow$ Poles for cavity + qubits,

The latter not observed unless two tones spectroscopy

$$S_{21} = \frac{(Q_l/|Q_c|)e^{i\phi}}{1 + 2iQ_l\left(f_0/f_c - 1 - \sum_{j=1}^{N} \frac{g_j^2/(2\pi f_c \Delta_j)}{\sqrt{1+4g_j^2 n_{ph}/\Delta_j^2}}\right)} \qquad (\omega_0 = 2\pi f_0, \omega_c = 2\pi f_c) \tag{8}$$

Next order equations: quantum radiation

$$(\partial_t + \gamma_c)\langle \hat{a}^\dagger \hat{a}\rangle = iB(e^{2i\theta}\langle \hat{a}^{\dagger 2}\rangle - e^{-2i\theta}\langle \hat{a}^2\rangle) \quad (9)$$

$$[\partial_t + 2i(\omega_s + B) + \gamma_c]\,\langle \hat{a}^{\dagger 2}\rangle = \frac{B}{i}e^{-2i\theta}(2\langle \hat{a}^\dagger \hat{a}\rangle + 1) \quad (10)$$

$$B = n_{ph}\frac{d\omega_s}{dn_{ph}} = \sum_{j=1}^{N} \frac{2\Delta_j g_j^4 n_{ph}/|\Delta_j|}{(\Delta_j^2 + 4g_j^2 n_{ph})^{3/2}} \quad (11)$$

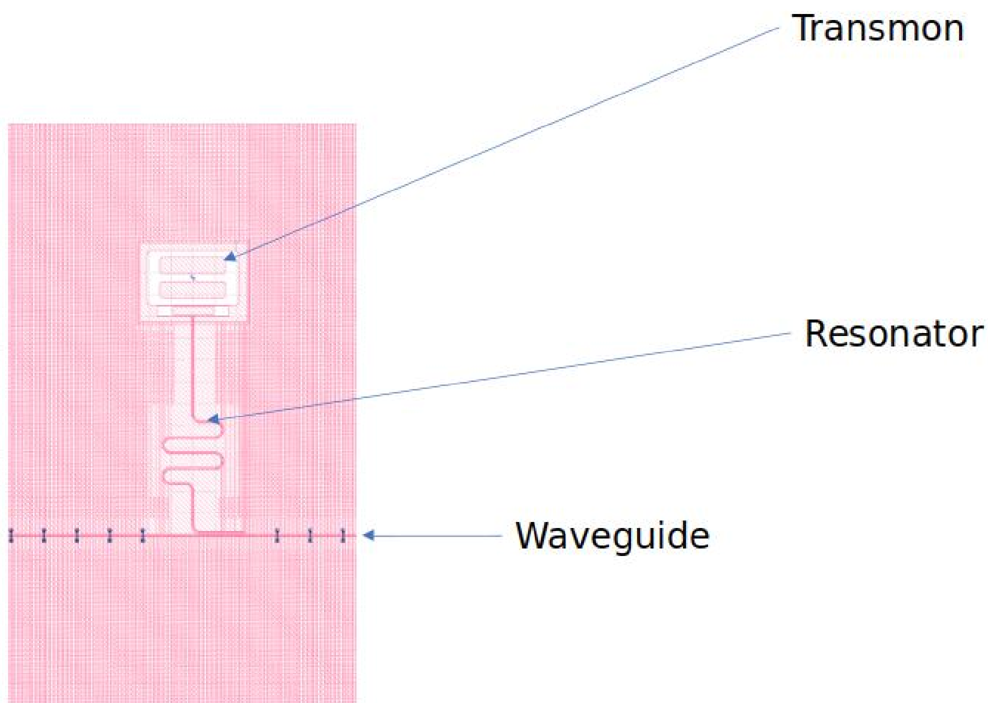
$$e^{i\theta} = \frac{\beta_0}{|\beta_0|} = \frac{\gamma_l/2 + i\omega_s}{\sqrt{\omega_s^2 + \gamma_l^2/4}} \quad (12)$$

The expressions for the time-dependant internal quality factor is:

$$Q_i(t) = \frac{\omega_0|\beta|^2}{iB(e^{2i\theta}\langle \hat{a}^{\dagger 2}\rangle - e^{-2i\theta}\langle \hat{a}^2\rangle)} \quad (13)$$

Fluorescence generates Q_i

Experiment in Iena: one qubit



Experiment in Iena: notch resonator

Probst et al. formula (https://arxiv.org/abs/1410.3365)

$$S_{21}^{notch} = a \exp(i\alpha - 2\pi i f_0 \tau)\,(1 - S_{21}^{ren}) \quad (14)$$

$$S_{21}^{ren} = \frac{(Q_l/|Q_c|)e^{i\phi}}{1 + 2iQ_l\left(f_0/f_c - 1 - \sum_{j=1}^{N} \frac{g_j^2/(2\pi f_c \Delta_j)}{\sqrt{1+4g_j^2 n_{ph}/\Delta_j^2}}\right)} \quad (15)$$

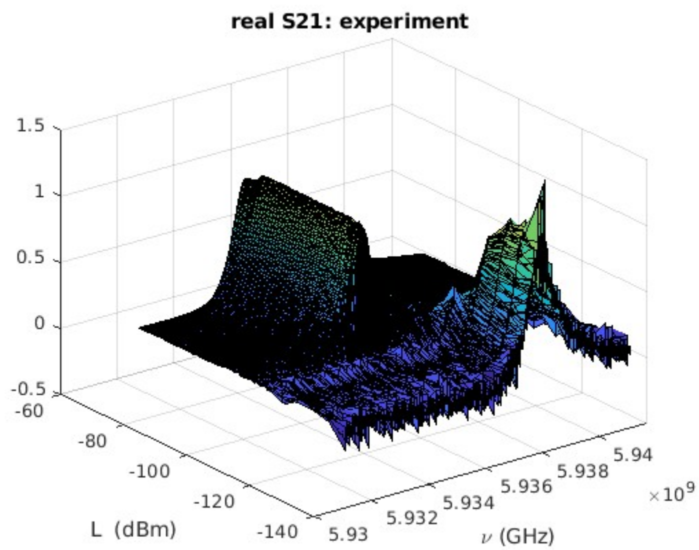
Attenuation factor a

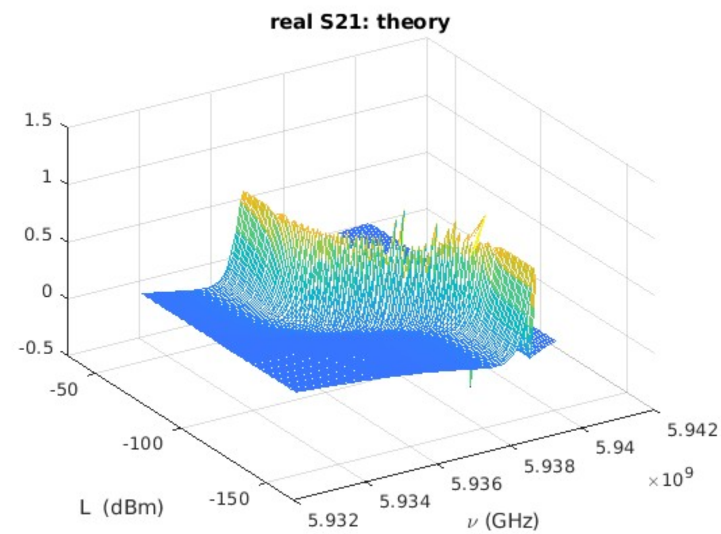
Experiment in Iena: one qubit

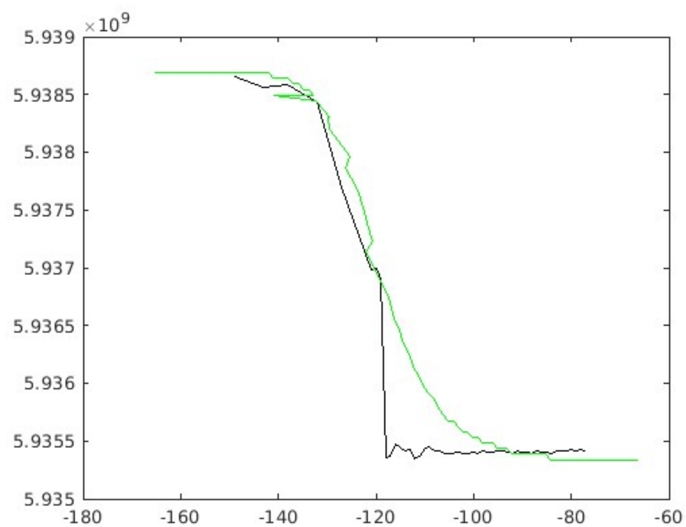
Sample number	Resonance freq, fr [GHz]	Internal quality, Qi	Coupling quality, Qc	f01 [GHz]	f12 [GHz]
1	6.3566	$248.924 \cdot 10^3$	$3.3070 \cdot 10^3$	5.538	5.307
2	5.837345	$855.94 \cdot 10^3$	$4.7691 \cdot 10^3$	7.561	7.152
3	5.3677	$215.35 \cdot 10^3$	$5.5984 \cdot 10^3$	5.72429	5.4917
4	4.92439	$348.647 \cdot 10^3$	$6.276 \cdot 10^3$	5.407	5.247
5	4.36753	$537.827 \cdot 10^3$	$10.492 \cdot 10^3$	5.1342	4.675
6	5.93532531	$47.642 \cdot 10^3$	$4.758 \cdot 10^3$	4.2495	4.0095

Very high internal quality factor $\Rightarrow$ quasi no losses

Experiment in Iena: one qubit (sample 6)

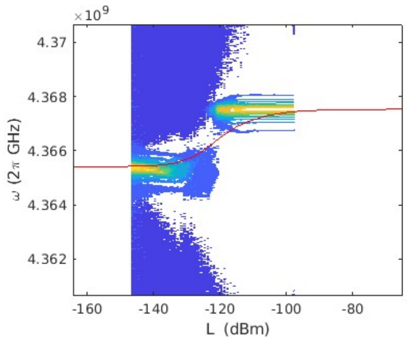


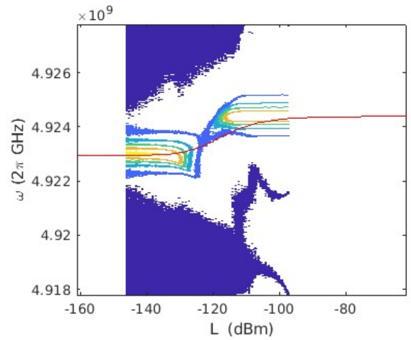


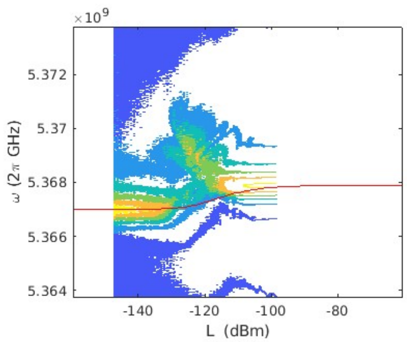


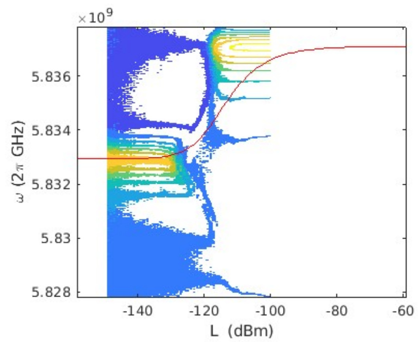
Experiment in Iena: one qubit (samples 1-5)

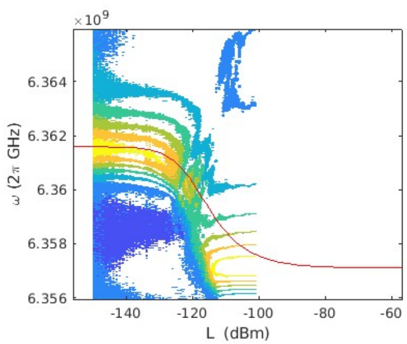
attenuations $a = 0.0147, 0.0155, 0.0137, 0.0112, 0.0102$





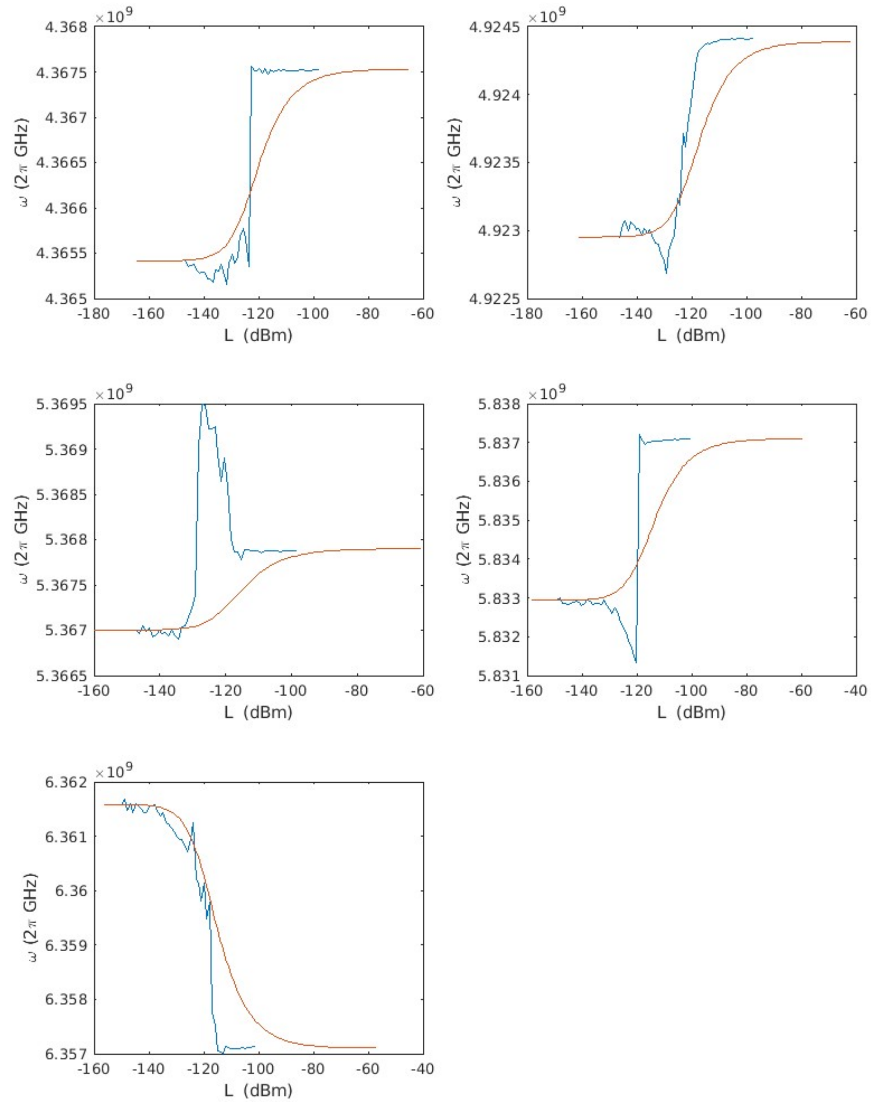






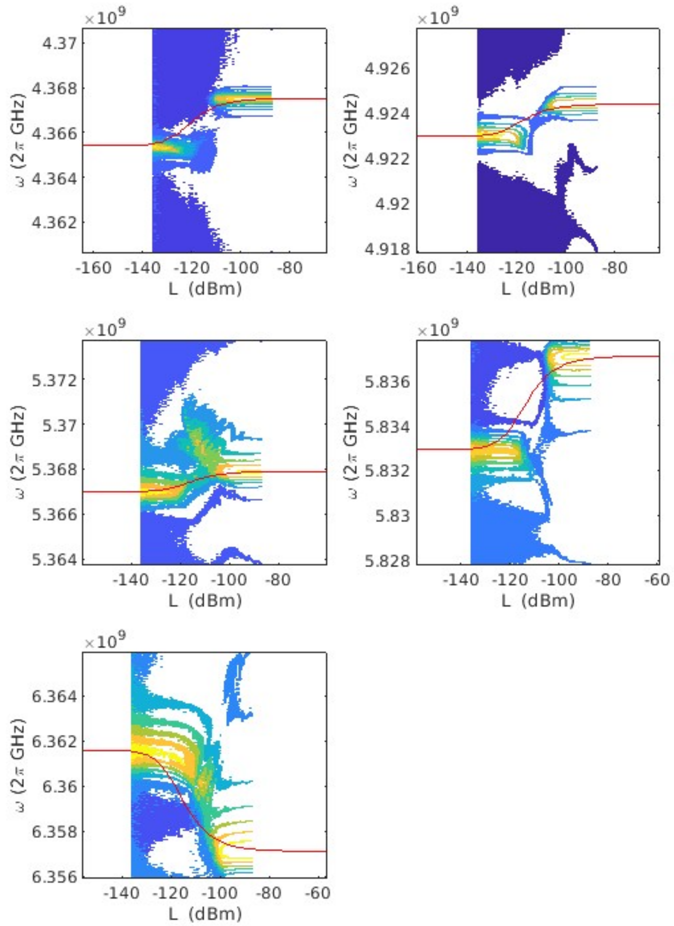
Experiment in Iena: one qubit

Noise in the signal ? too preliminary



Experiment in Iena: one qubit

Less attenuation: controlled parameter $a = 0.05 \to$ curves move to lower power



Experiment in Iena: five qubits in embedded resonator

Assumption: qubit with same shift and coupling

Fitting Parameters: $\omega_c = 2\pi\, 6.61\text{GHz}$,

$$\omega_s(n_{ph} \to \infty) - \omega_s(n_{ph} = 0) = 2\pi\, 10\text{MHz},$$

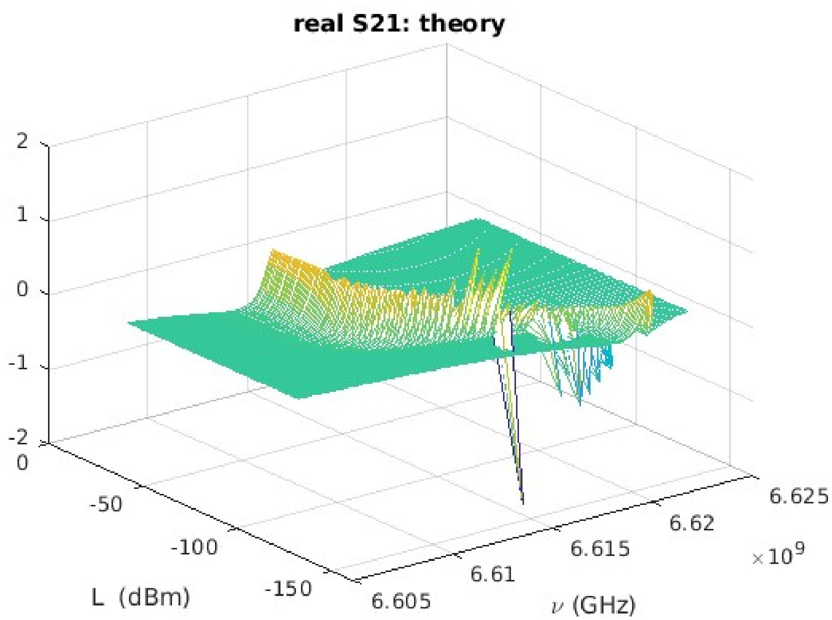
$$\gamma_c = 2\pi\, 400\text{kHz},$$

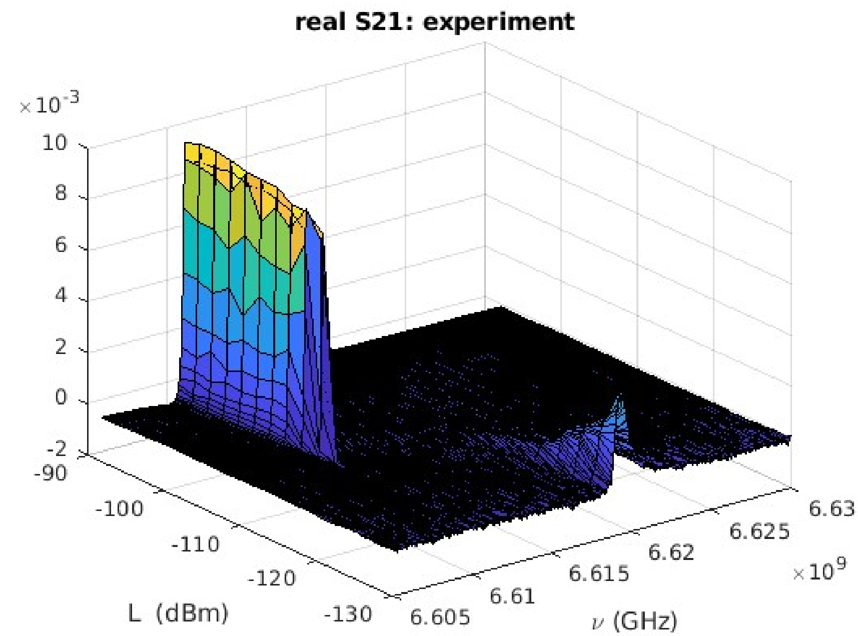
$$\Delta_j = 2\pi\, 177\text{MHz},$$

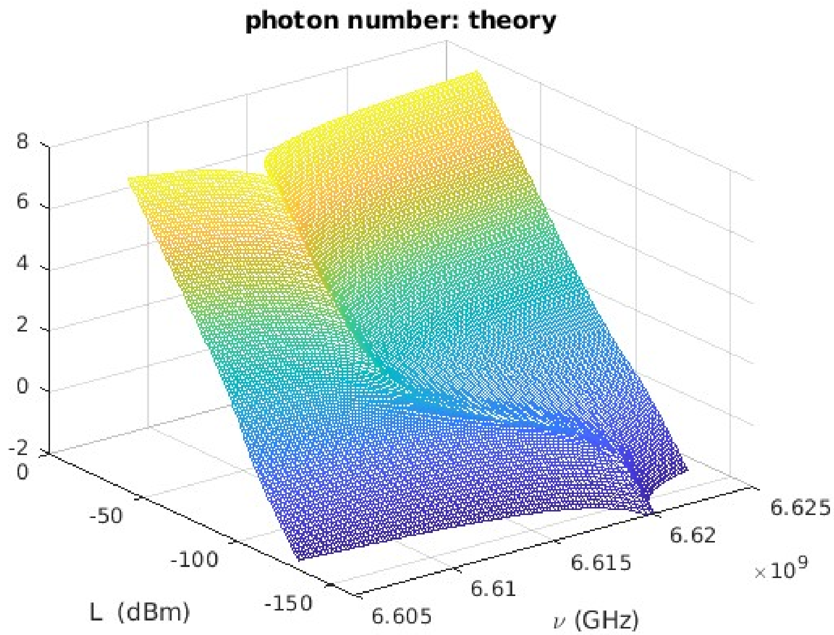
$$g_j = 2\pi\, 40\text{MHz},$$

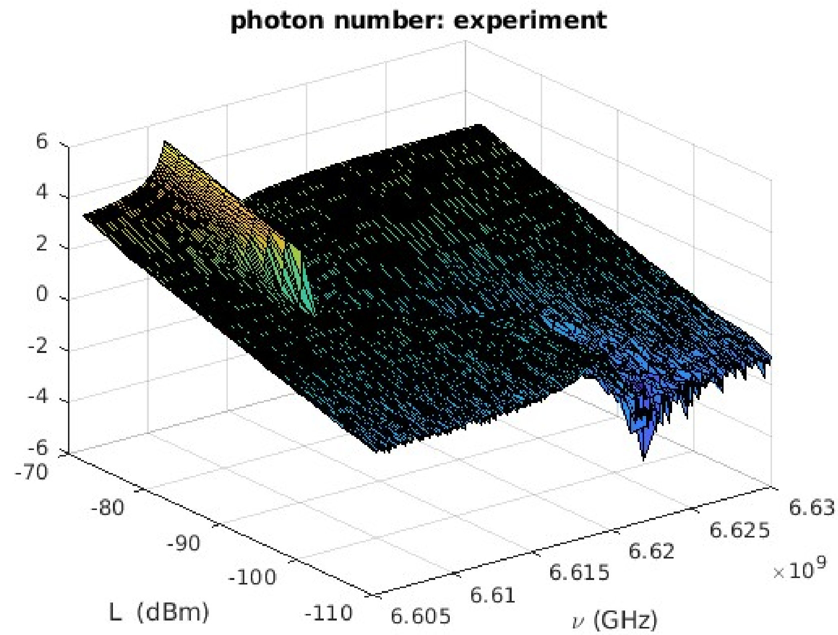
Other experiments with shift of 0.1MHz provides less non linearity

$\mathrm{Re}S_{21}$ and photon number



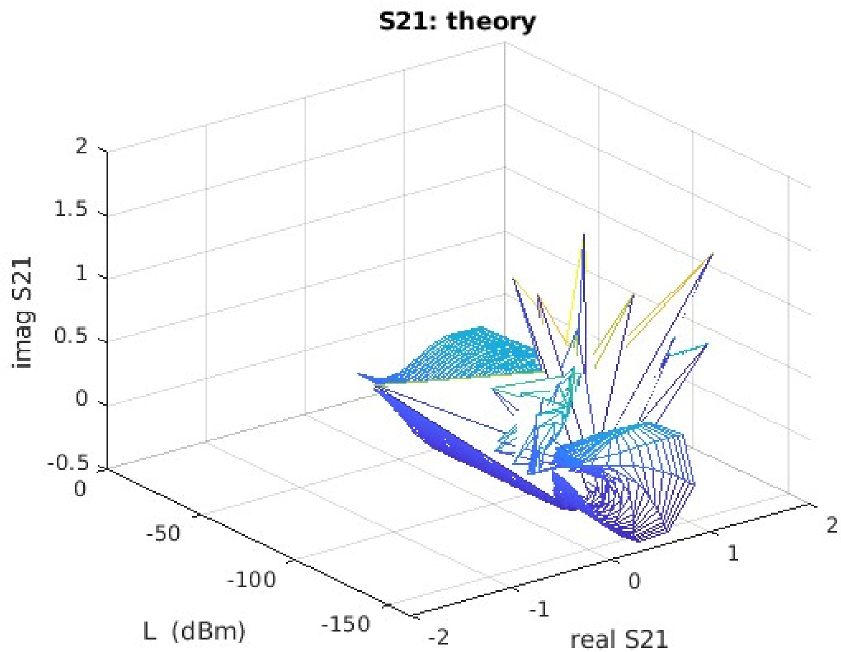


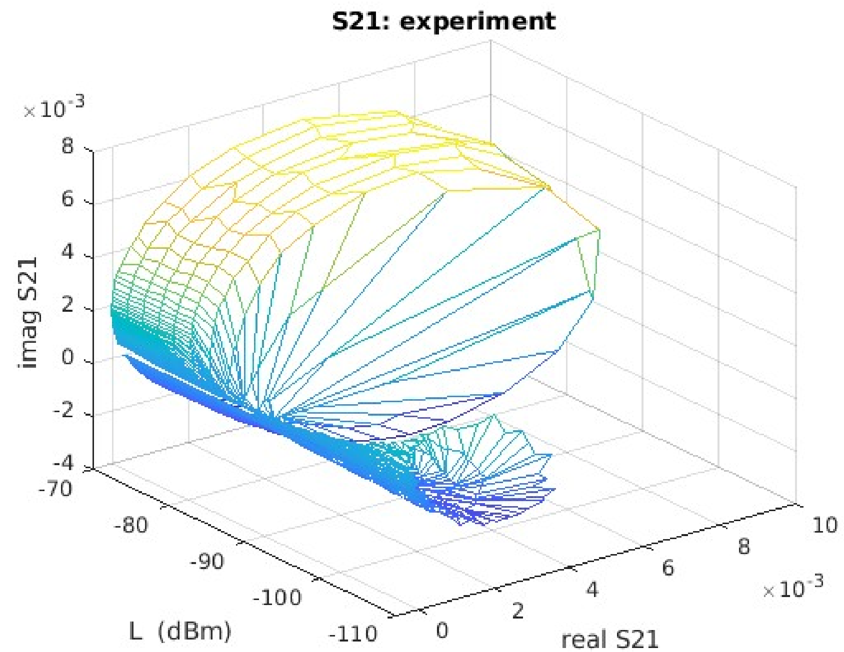




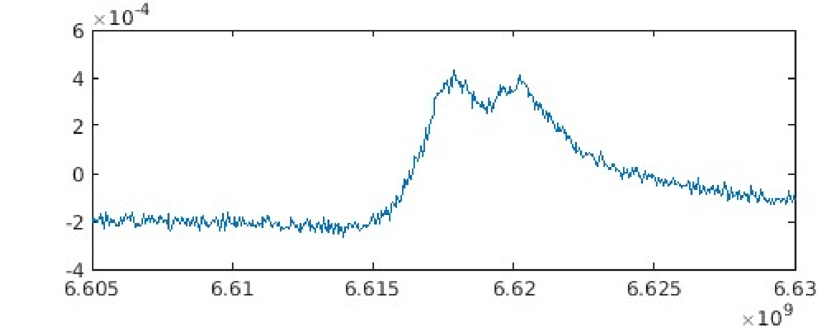
Bistability !

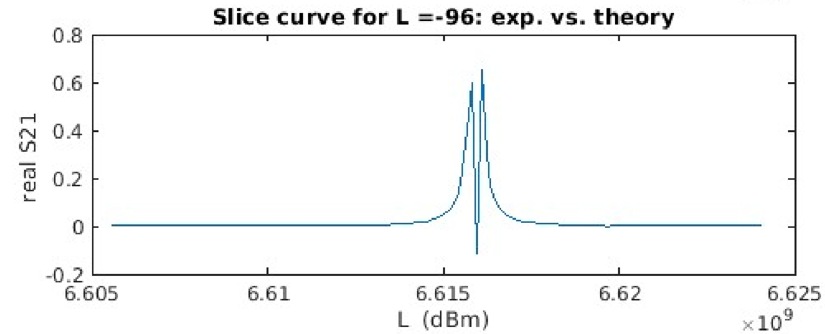
Lissajous curves

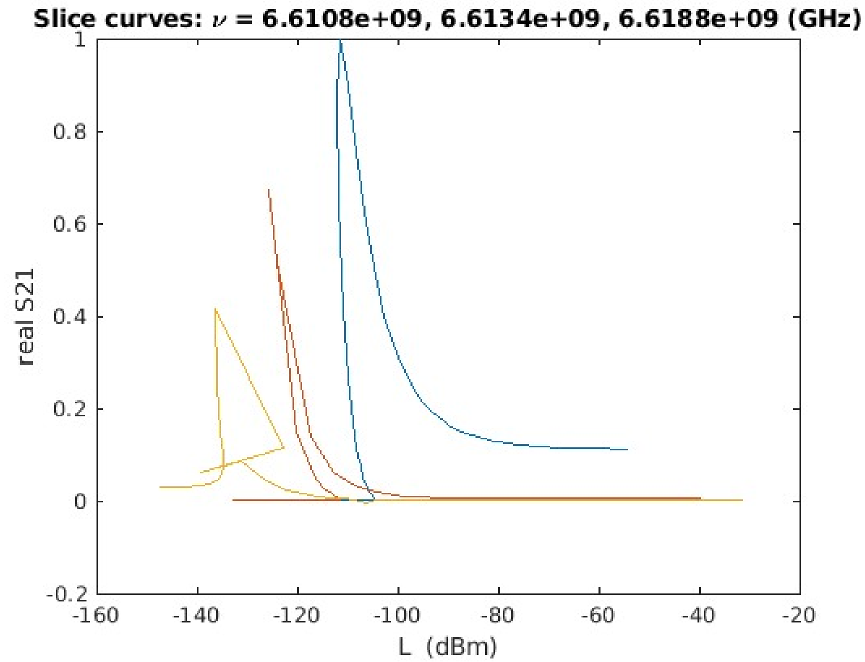




Double peak and bistability

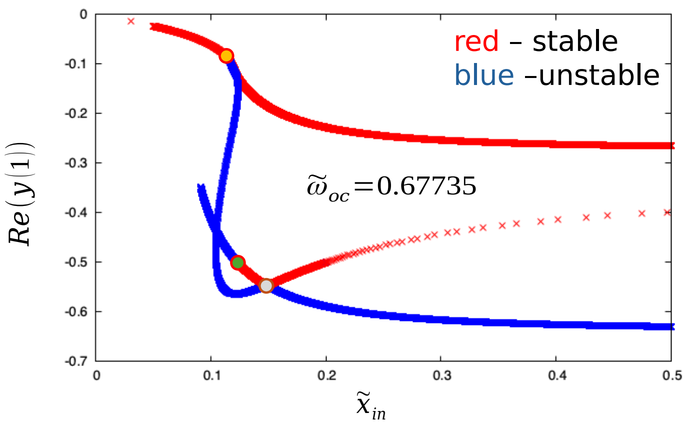






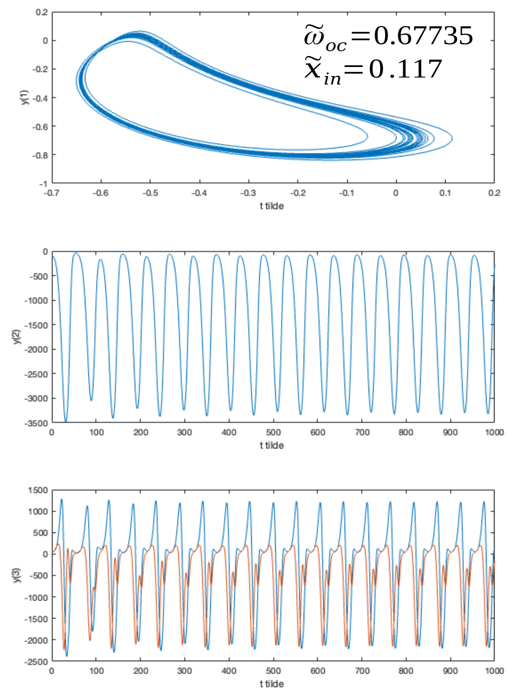
Duffing instability

Instability analysis with Sacha Balanov



- Andronov-Hopf bifurcation
- saddle-node bifurcation
- transcritical bifurcation

possibly chaos (see matlab code)



Summary

Theoretical description of a single photon detector: QND process using qubits

- Quantum electrodynamics: Explicit S_{21} in presence of cavity
- Potential use for reservoir computing
- Th-exp comparison: Power dependent Stark Shift: hysteresis and bistability
  potential new physics: QUANTUM INSTABILITY!
- Improvements from experiments (more data) or from theory (better fitting)

  Additional contribution to Q_i ??? qubit relaxation T_1, T_2
- Perspective: higher moment development for weaker detuning

  Towards the huge Fock space!

References

- P. Navez et al., J. Appl. Phys. **133**, 104401 (2023)
- Alessandro D'Elia et al., IEEE Trans. on Appl. Superconductivity **33** (1) (2023)
- P. Navez et al., Phys. Rev. B **103**, 064503 (2021)

  Heisenberg limit $\rightarrow$ noise suppression : $1/N \rightarrow 1/N^2$