Manybody Systems & Quantum Simulation

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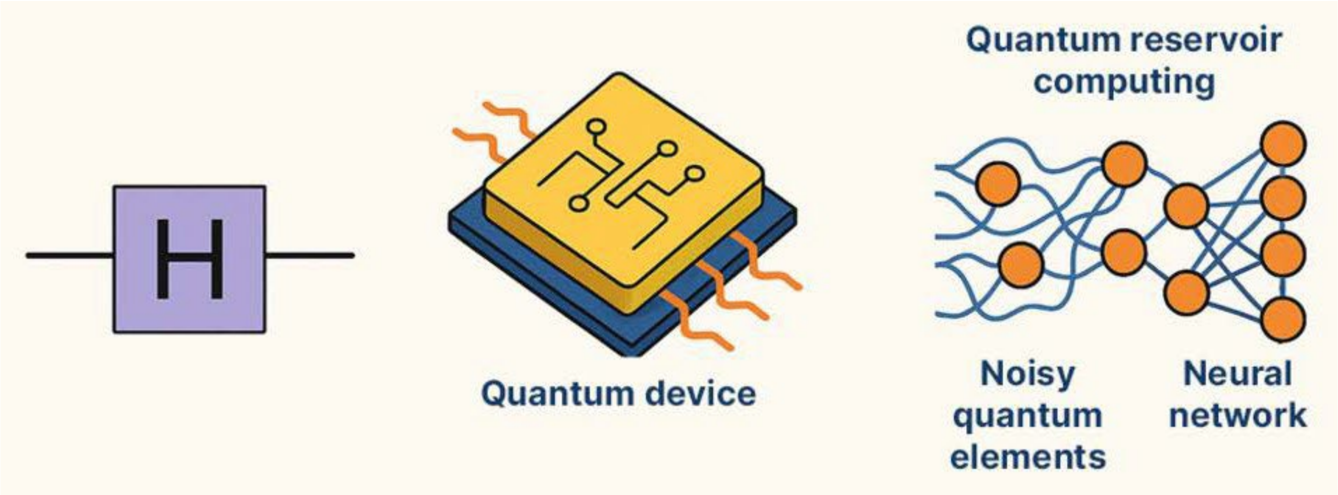
Tampere University & Aalto University



Summer School on Quantum Reservoir Computing, Quantum Devices and Related Technologies

Aalto University, August 2025

Outline

- Overview of manybody systems
- The type of systems, key features, main theories/paradigms
- Quatum-inforamtic take on manybody systems
- Simulating manybody systems (best use case for QC?)
- Overview of quantum simulation
- Practical use cases (modern manybody era)
- Far from equilibrium, disoredered, interacting systems
- Summary

Manybody physics in one-page!

Atomic conformations of molecules

Understanding chemical reaction (rates, energy scales, ...)

Electronic, mechanical, and optical properties of solids

Phase transitions & strongly correlated physics

Magnetism and ordered phases (ferromagnetism, antiferromagnetism)

Superconductivity (BCS vs unconventional)

Topological order (fractional quantum Hall, fractional & non-abelian statistics)

Manybody physics in one-page!

$$i\hbar \frac{\partial}{\partial t} |\Psi > \; = \mathcal{H} |\Psi >$$

$$\mathcal{H} = -\sum_{j}^{N_e} \frac{\hbar^2}{2m} \nabla_j^2 - \sum_{\alpha}^{N_i} \frac{\hbar^2}{2M_\alpha} \nabla_\alpha^2$$

$$-\sum_{j}^{N_e}\sum_{\alpha}^{N_i} \frac{Z_\alpha e^2}{|\vec{r}_j - \vec{R}_\alpha|} + \sum_{j \ll k}^{N_e} \frac{e^2}{|\vec{r}_j - \vec{r}_k|} + \sum_{\alpha \ll \beta}^{N_j} \frac{Z_\alpha Z_\beta e^2}{|\vec{R}_\alpha - \vec{r}_\beta|}$$

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Manybody physics in one-page!

Effective model
Empirical input for guidance
Approximate calculations

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Understanding chemical reaction (rates, energy scales, ...)
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Maybe one more page is needed!

The *emergent* **physical phenomena regulated by** *higher organizing principles* **have a property, namely their** *insensitivity to microscopics.*

Phillip W. Anderson

Emergence

Steven Weinberg

Reductionism

[Anderson '72, Laughlin & Pines '00]

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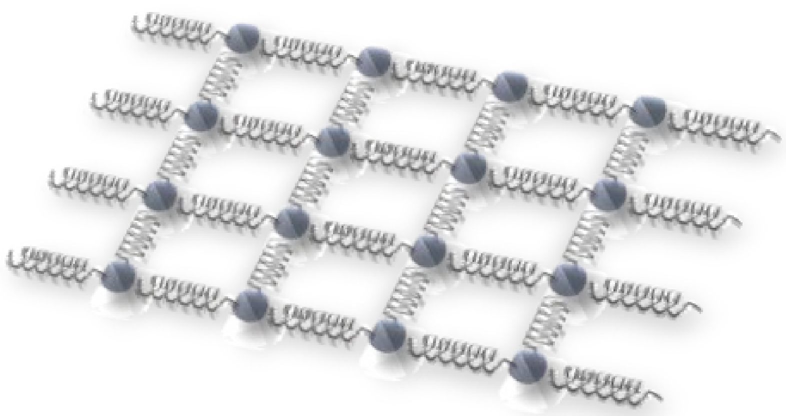

Phillip W. Anderson

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Reductionism

One main purpose of CMP is to study

The elementary excitations

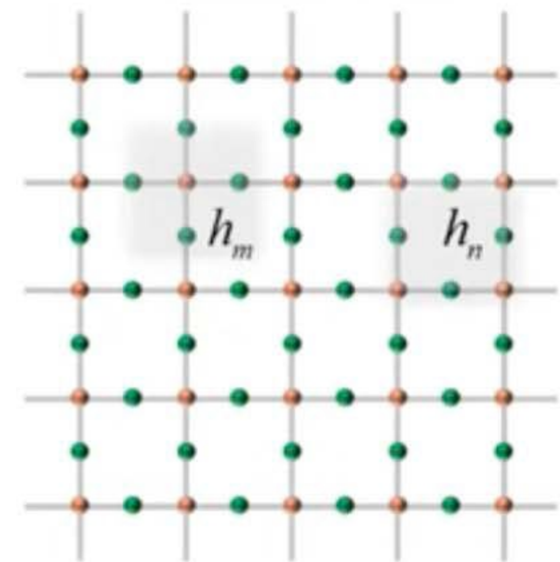
Interactions
Particle statistics
Disorder
Far-from-equilibrium

[Anderson ’72, Laughlin & Pines ’00]

Key problems/properties in manybody systems

- Ground-state & lowest-energy properties
- **Dynamics / time evolution**

$$|\epsilon(t)\leftrightarrow = e^{\rightarrow iHt}|\epsilon_0\leftrightarrow$$

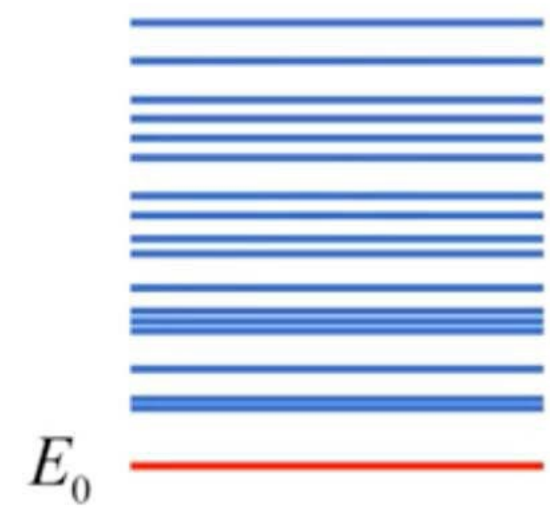


$$H = \sum_{n=1}^{N} h_n$$

- Excited states, quasiparticles, response functions
- **Thermodynamics & statistical properties**

$$\omega_\omega = \frac{e^{\rightarrow\omega H}}{Z}$$

$$Z = Tr(e^{\rightarrow\omega H})$$



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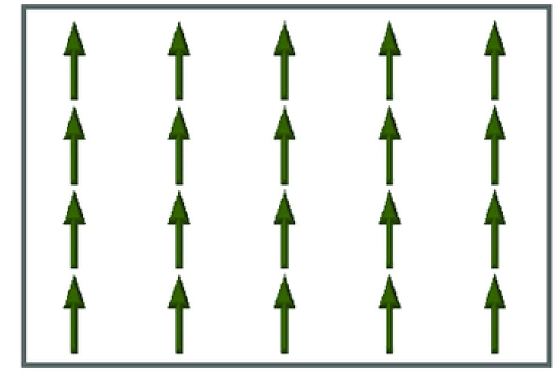

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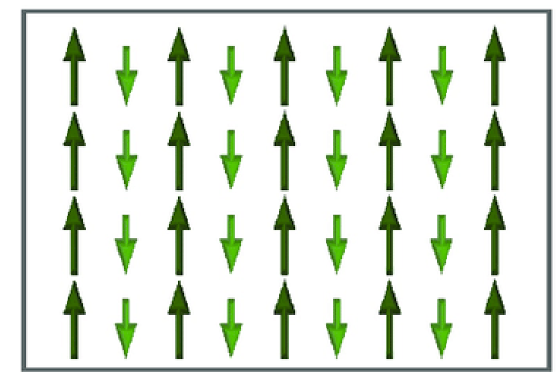

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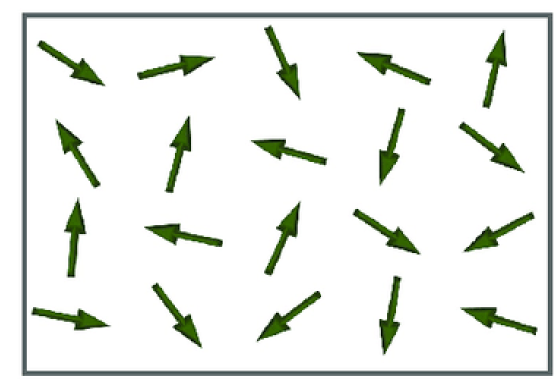
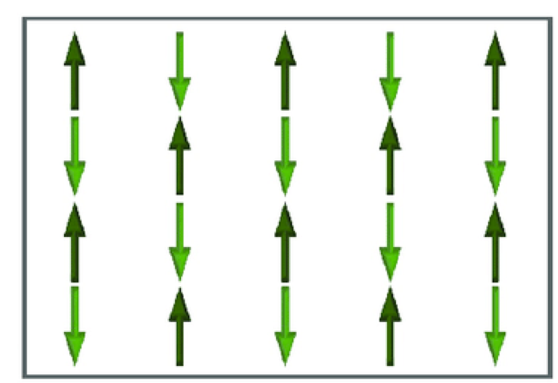


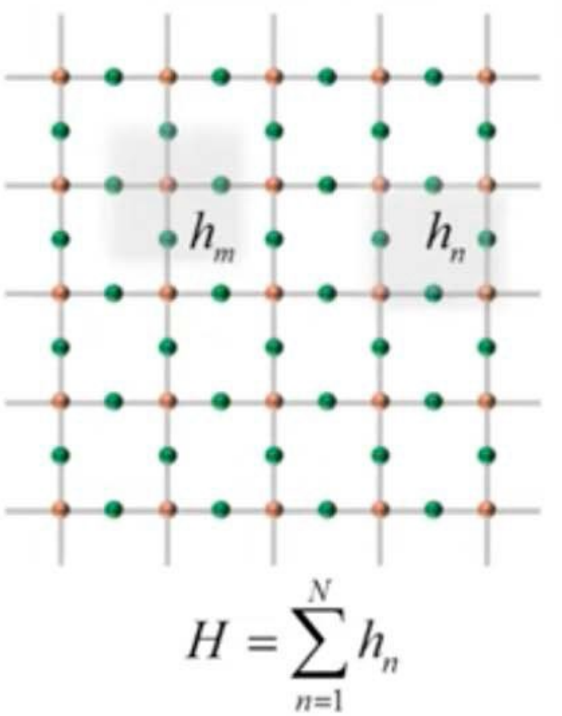


- Correlation functions and order parameters
- Emergent phenomena:

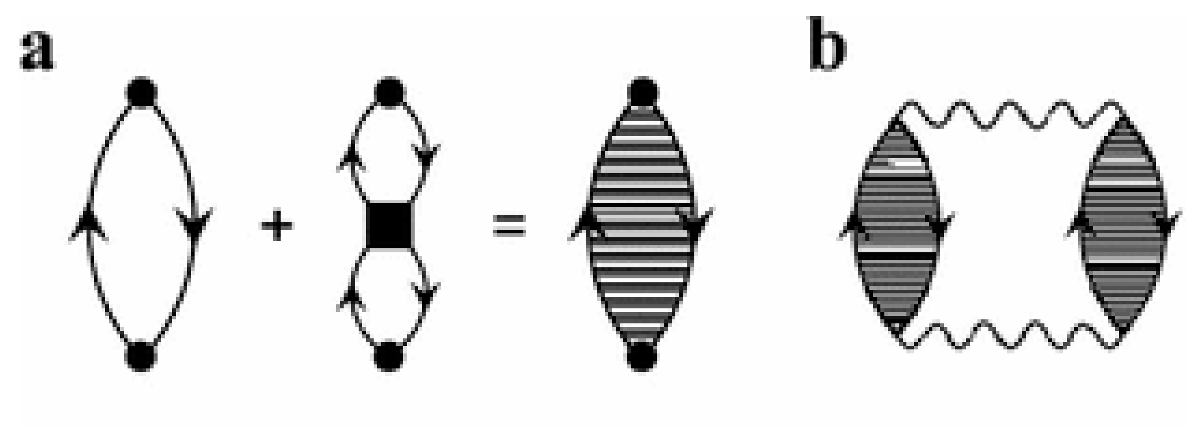
  superconductivity, magnetism, critical phenomena

Paradigms, techniques, and guiding rules

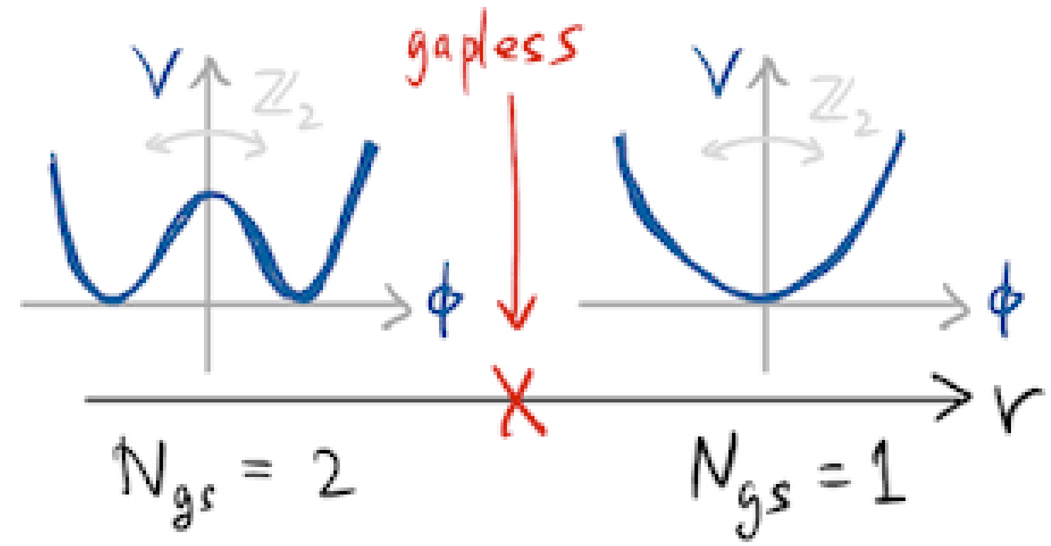
- Effective models



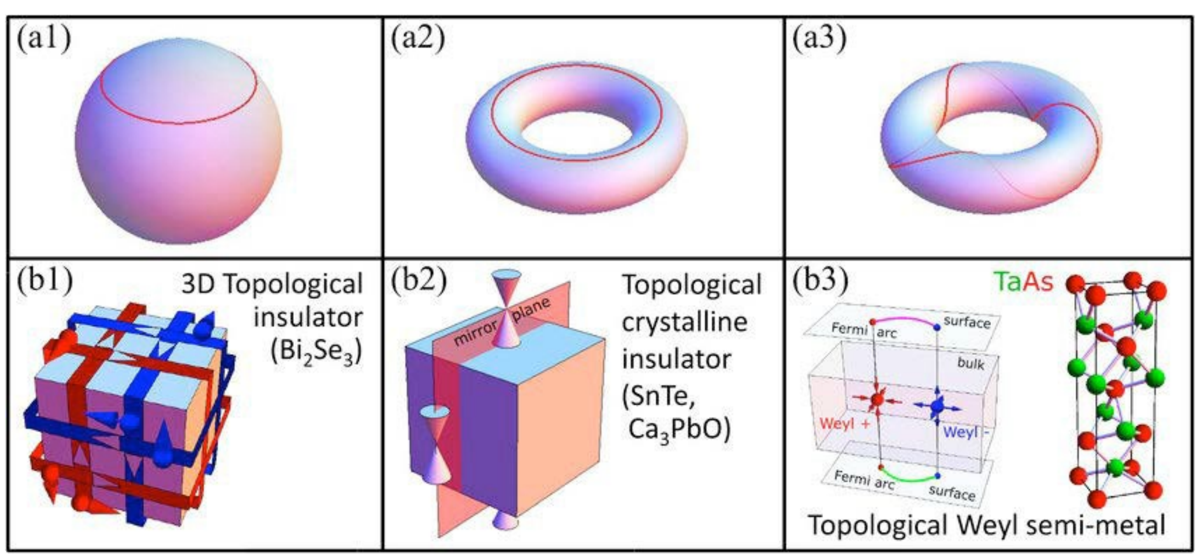
- Approximations, perturbation theory



- conservation law, symmetries, & breaking them



- topology and quantum order



Major breakthroughs in understanding manybody systems

Bloch's Theorem and band theory (1928)

Quasiparticle Concept (Landau, 1940s–50s)

Fermi Liquid Theory (Landau, 1956)

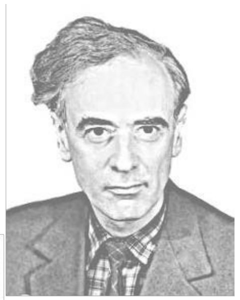
Field theoretic appraoch and developement of manybody theory (1950s)

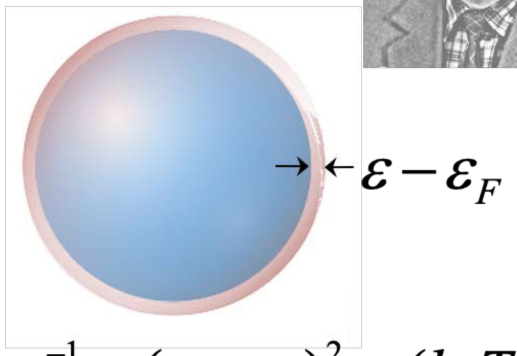
BCS Theory of Superconductivity (Bardeen, Cooper, Schrieffer, 1957)

Symmetry breaking, Renormalization Group (1950s to 1970s)

Strongly Correlated Systems (mostly after 1980s)

Available phase space for scattering processes near the Fermi surface is very small

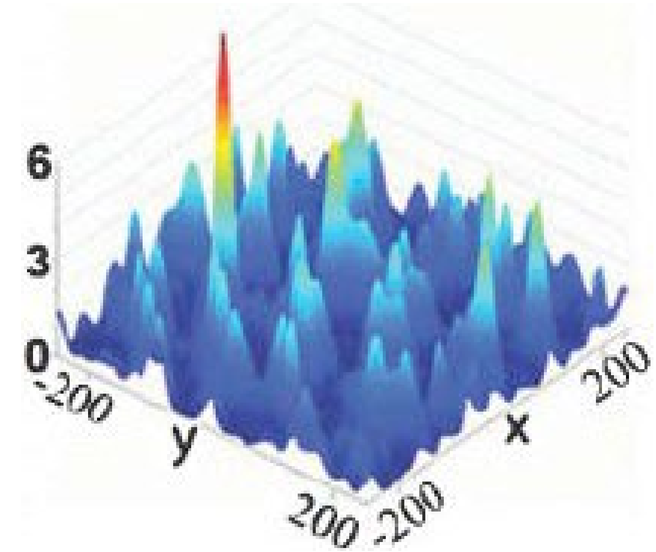




$$\tau^{-1} \sim (\varepsilon - \varepsilon_F)^2 \sim (k_B T)^2$$

No perturbation is possible

Major breakthroughs in understanding manybody systems



Anderson Localization (1958):

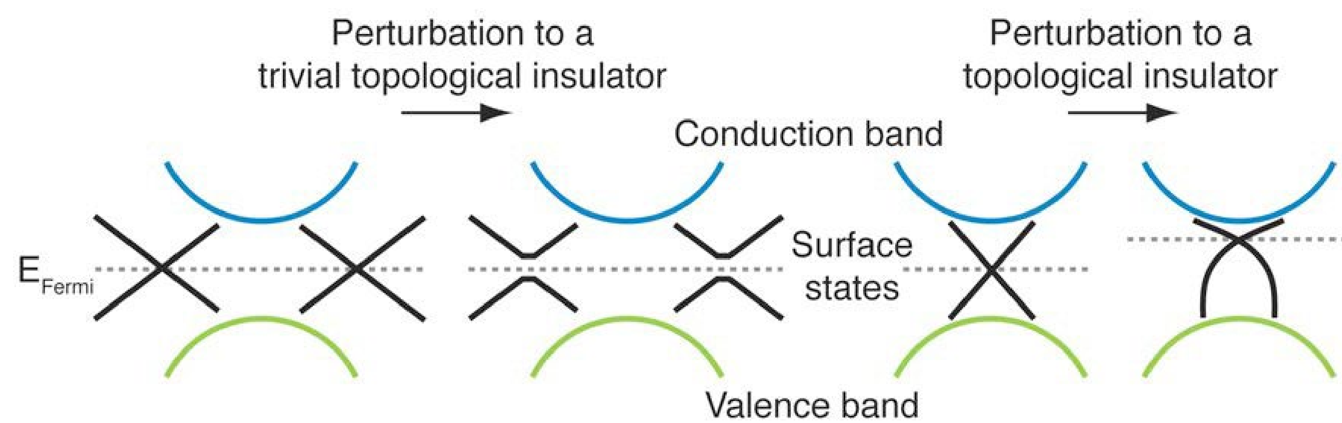
Many-Body Localization (2000s):

Hubbard Model & Strongly Correlated Systems (mistly after 1980s)

Quantum Hall Effect (Integer, 1980; Fractional, 1982)

Topological Phases of Matter (2000s)

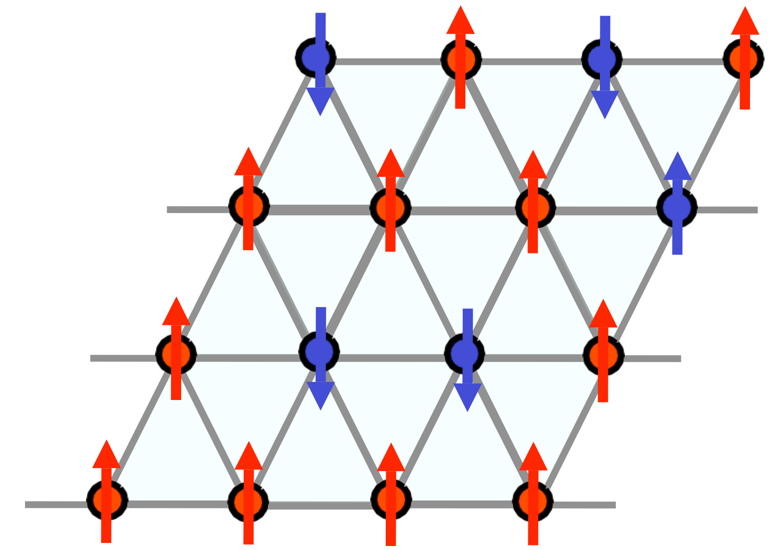
Nonequilibrium & Floquet Phases (2010s)



Manybody Hilbert space

$$State = \Psi = \sum_x \alpha_x \left| x \right\rangle = \begin{pmatrix} \alpha_{0000} \\ \alpha_{0001} \\ . \\ . \\ \alpha_{1111} \end{pmatrix}$$

all n-bit strings



Hilbert space associated with n-qubit system is

$$C^{2^n}$$

Exponentially large Hilbert spaces

Entanglement

Quantum complexity

Hilbert space dimension of even 300 qubits already surpassses the number atoms in the universe!

Particle statistics & Fock space

$$\Psi(x_1, \ldots x_n) = \frac{1}{\sqrt{n!}} \begin{vmatrix} \psi_1(x_1) & \cdots & \psi_n(x_1) \\ \vdots & \ddots & \vdots \\ \psi_1(x_n) & \cdots & \psi_n(x_n) \end{vmatrix}_\nu$$

$$F_\nu(H) = \bigoplus_{n=0}^{\infty} S_\nu H^{\otimes n} = \mathbb{C} \oplus H \oplus (S_\nu (H \otimes H)) \oplus (S_\nu (H \otimes H \otimes H)) \oplus \cdots$$

$(\nu = +)$ bosonic $\qquad (\nu = -)$ fermionic

$$\hat{a}_i |\ldots, n_i, \ldots\rangle = \sqrt{n_i}\, |\ldots, n_i \quad 1, \ldots\rangle$$

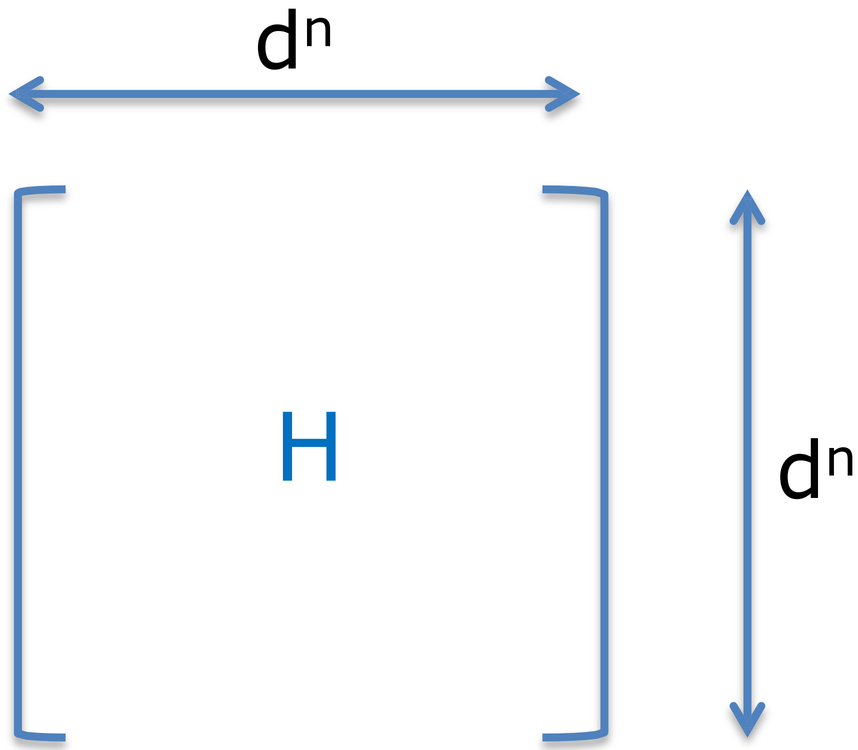
$$\hat{a}_i^\dagger |\ldots, n_i, \ldots\rangle = \sqrt{n_i + 1}\, |\ldots, n_i + 1, \ldots\rangle$$

$$[a_i, a_j^\dagger]_\nu = \delta_{ij}$$

$$[a_i, a_j]_\nu = 0$$

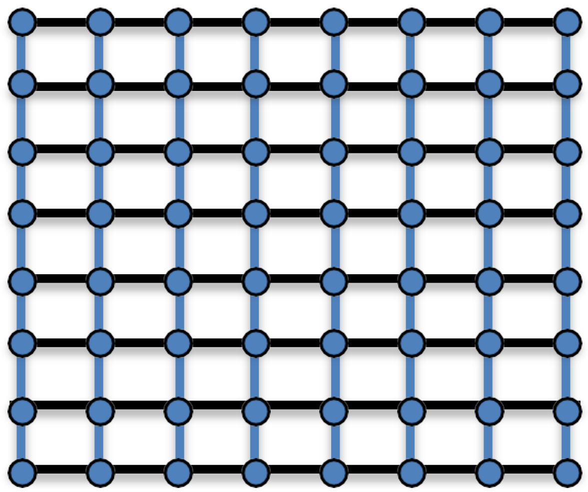
Manybody operators and Hamiltonians

Hamiltonian ***H***, d^n x d^n Hermitian matrix



Condensed matter physics: most of the interesting physics determined by ground and low-energy states of ***H***

(Common) Manybody problem



$$H = \sum_i H_i$$

Each term H_i is $d^2 \to d^2$, corresponding to a ***local two-particle*** term

Note: each term acts on the entire Hilber space meaning that:

$$H_i \equiv H_i \otimes I_{d^{n-2}}$$

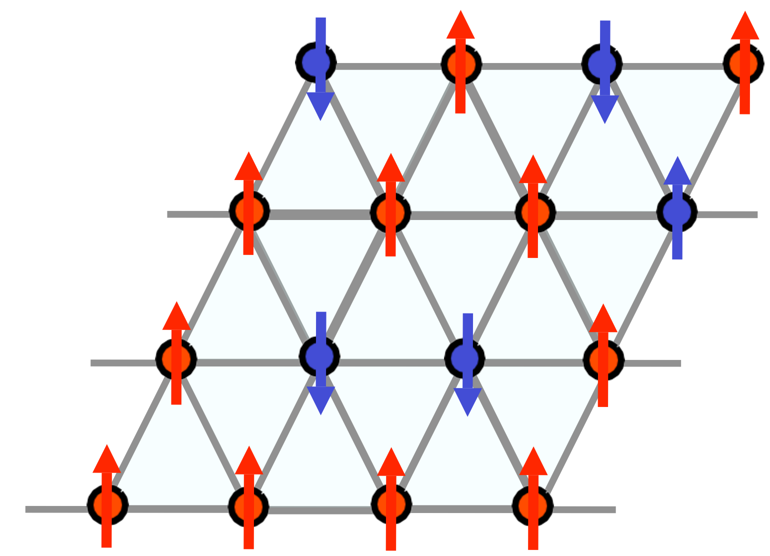
Given compact representation of the terms H_i,

can we hope for compact descriptions of ground state?

Important examples

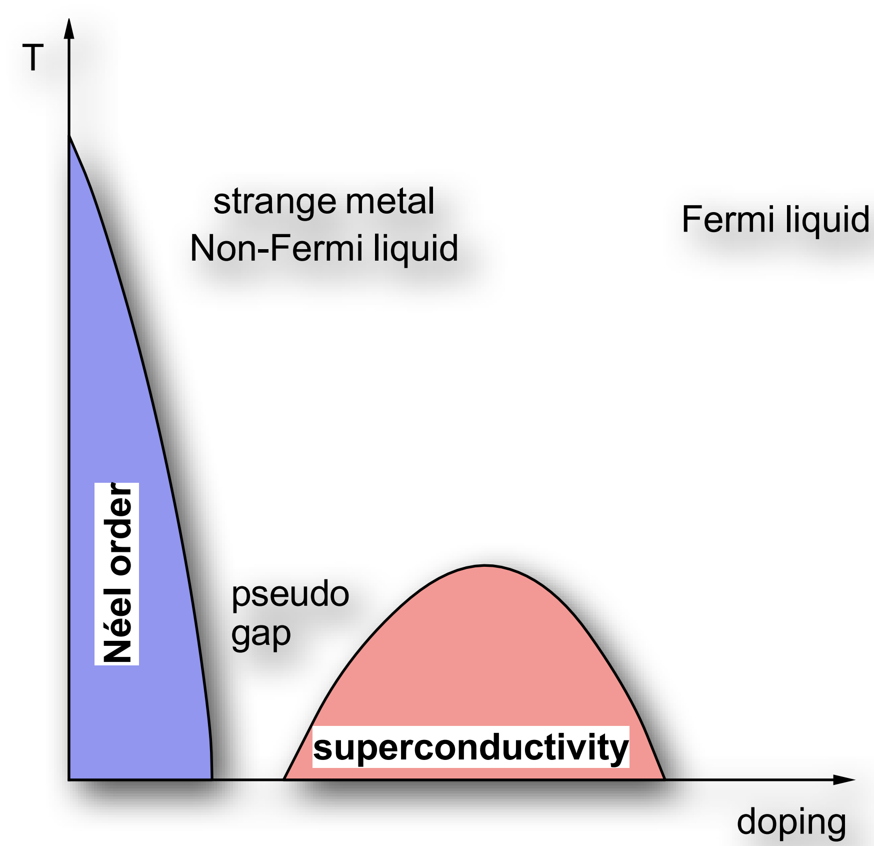
Heisenberg model

$$H_{\text{Heis}} = J \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j$$



Hubbard model

$$H_{\text{FH}} = -t \sum_{\langle i,j \rangle, \sigma} \left(c^{\dagger}_{i\sigma} c_{j\sigma} + c^{\dagger}_{j\sigma} c_{i\sigma} \right) + U \sum_{i} n_{i\uparrow} n_{i\downarrow}$$



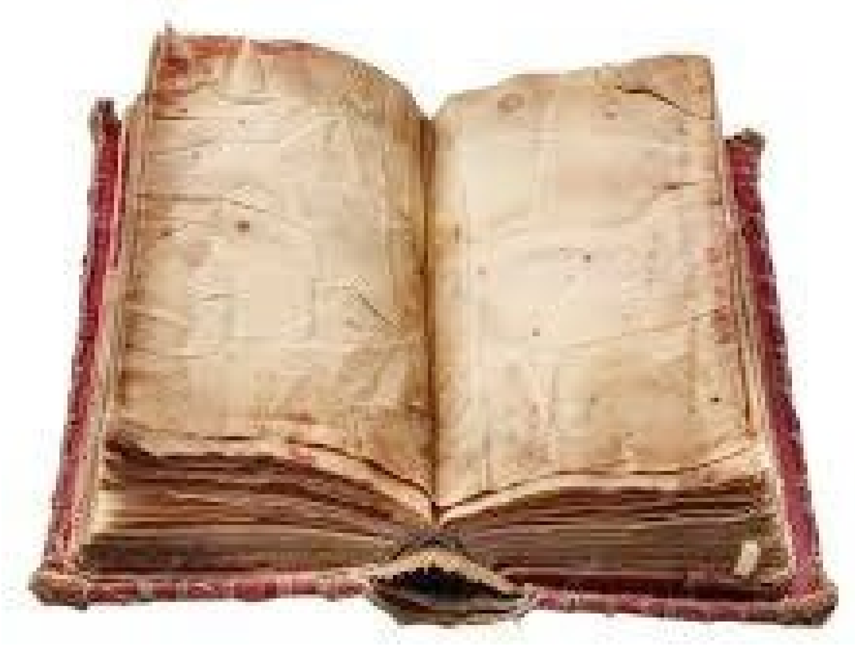
First break

Elephant in the room: Entanglement

$$|\psi\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$

Multi-particle states generally cannot be described by describing state of each particle.

Elephant in the room: Entanglement



Information in an entangled "quantum book" is encoded in correlations of "pages."

Reading pages individually does not access to the encoded information.

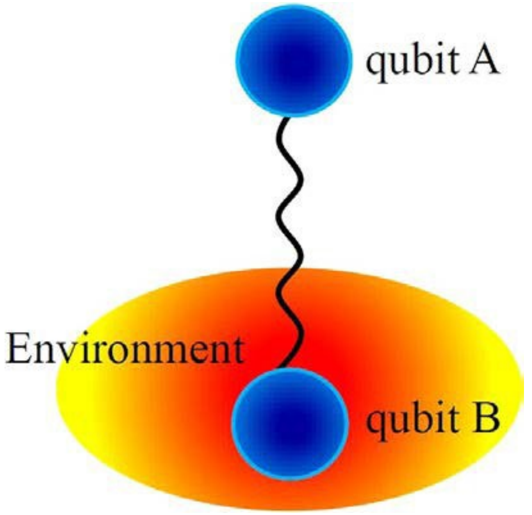
$$|\psi\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$

Multi-particle states generally cannot be described by describing state of each particle.

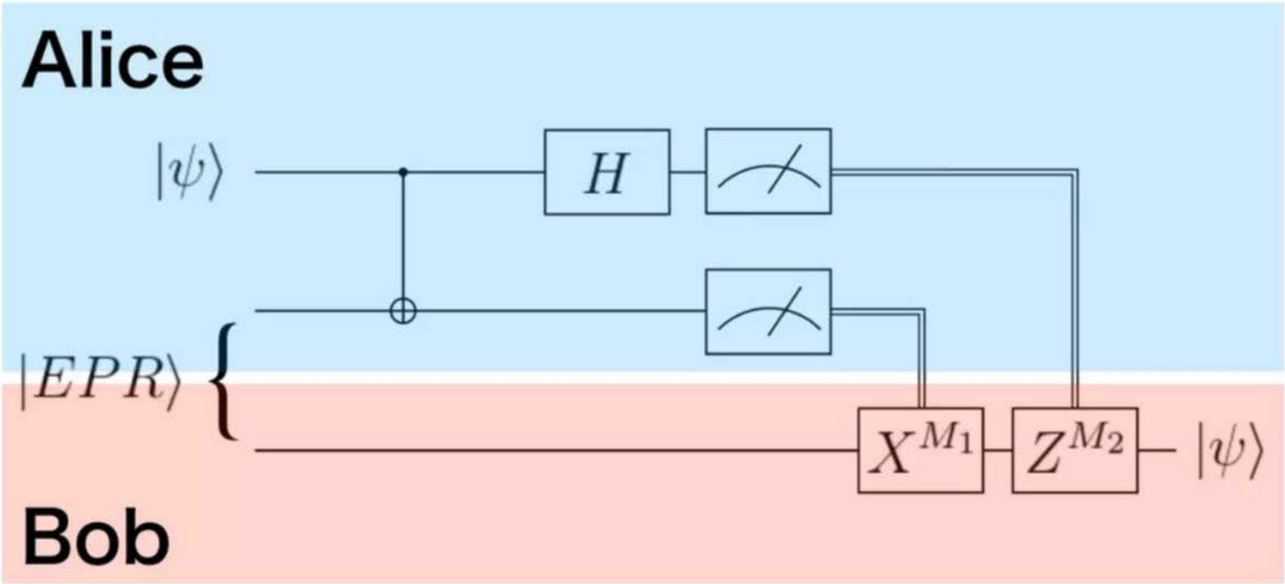
Entanglement: the curse and the blessing

The curse:

- Fragile
- Hard to probing
- Source of decoherence

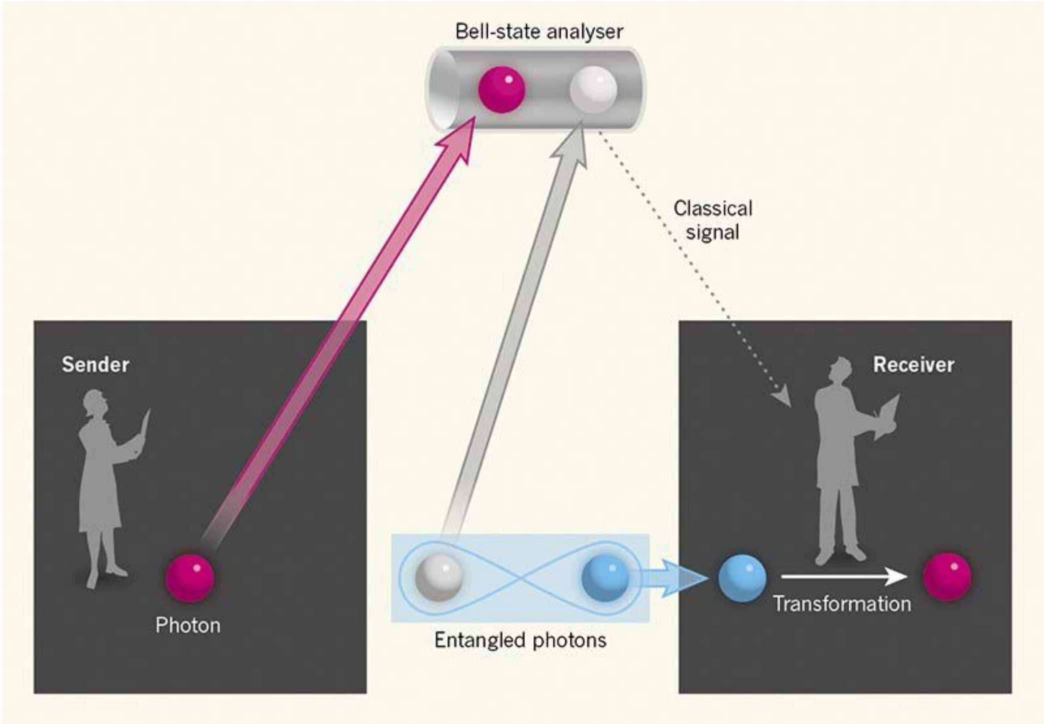


Entanglement: the curse and the blessing

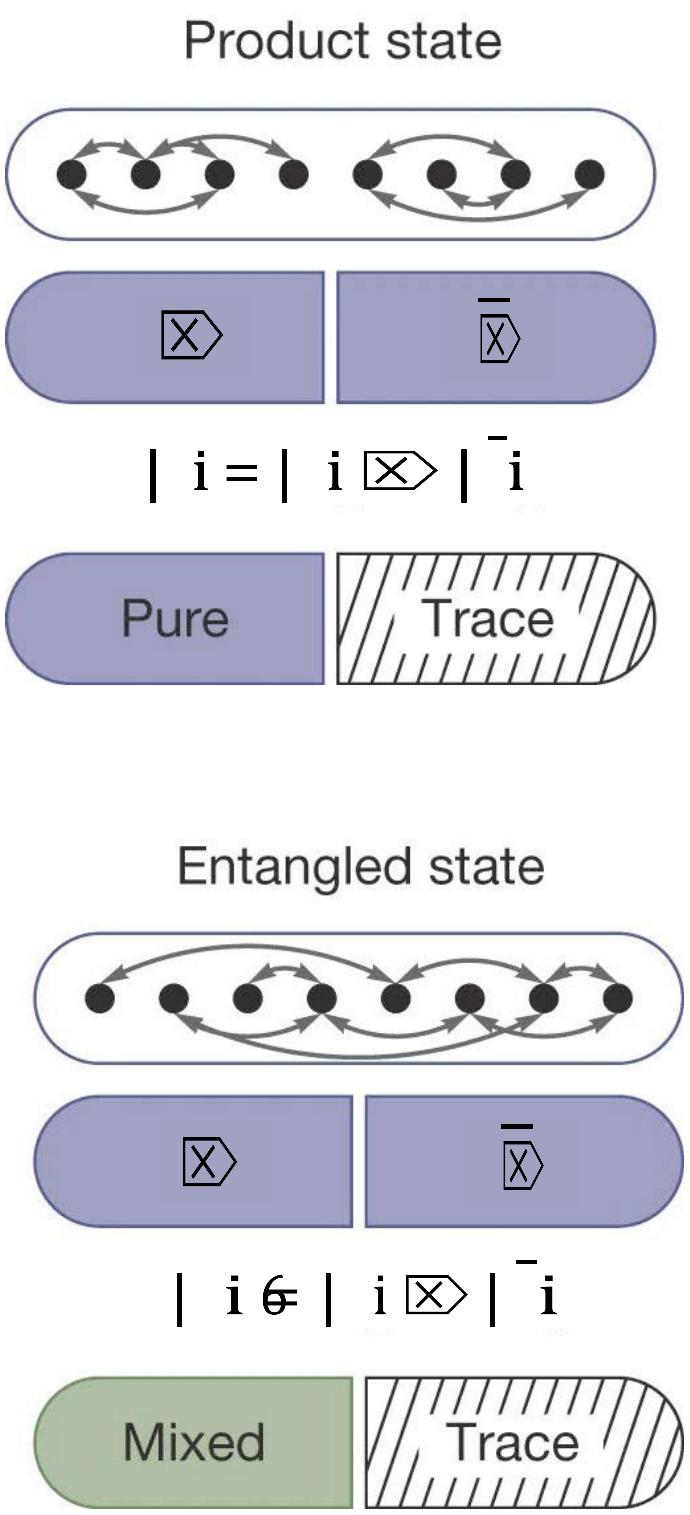


The blessing:

- Ubiquitous quantum source
- Quantum communication enabler
- Quantum computing powerhouse
- Quantum error correction



Manybody entanglement



Islam et al., Nature (2015)

Manybody entanglement

Schmidt decomposition

$$| \ \ i = \sum_i {}^{1/2}_{i} \ | \ _i i \boxtimes | \bar{\ }_i i$$

Reduced density matrix

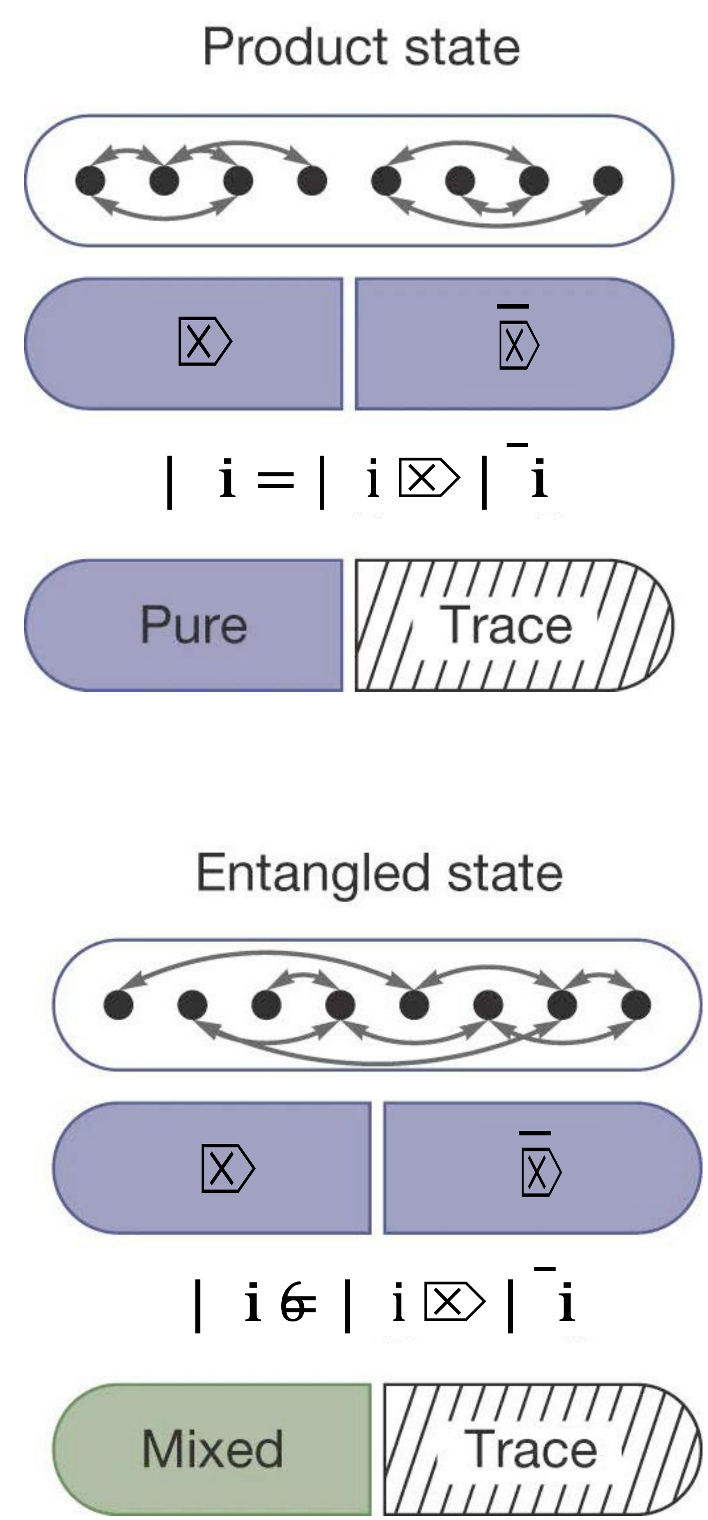
$$\dashrightarrow_{\boxtimes} = \mathrm{Tr}_{\bar{\boxtimes}} \dashrightarrow_{\mathrm{full}} = \mathrm{Tr}_{\bar{\boxtimes}} | \ \ ih \ \ | = \sum_i A_i \, |A_i i h A_i|$$

Entanglement entropies

$$S_{vN} = -\mathrm{Tr} \dashrightarrow_{\boxtimes} \ln \dashrightarrow_{\boxtimes} \qquad S_2 = -\ln \mathrm{Tr}(\dashrightarrow^2_{\boxtimes})$$

Kitaev, Preskill, PRL (2006); ...

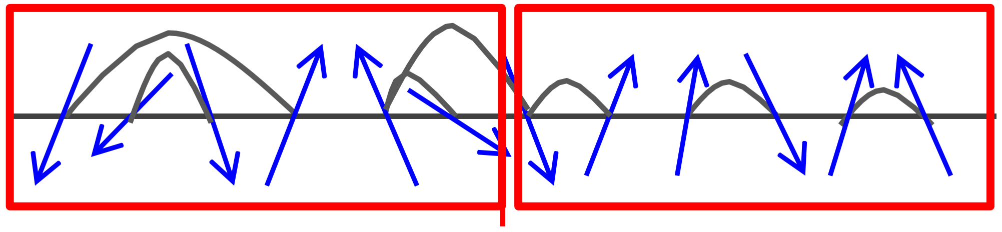
Li, Haldane, PRL (2008);
Nussinov, Ortiz, Ann Phys (2009);
Pollmann et al., PRL (2010); ...



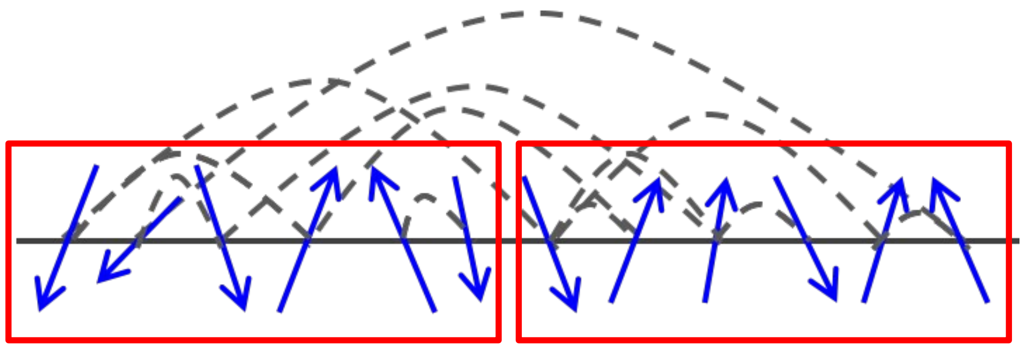
Islam et al., Nature (2015)

Area- & volume-law states

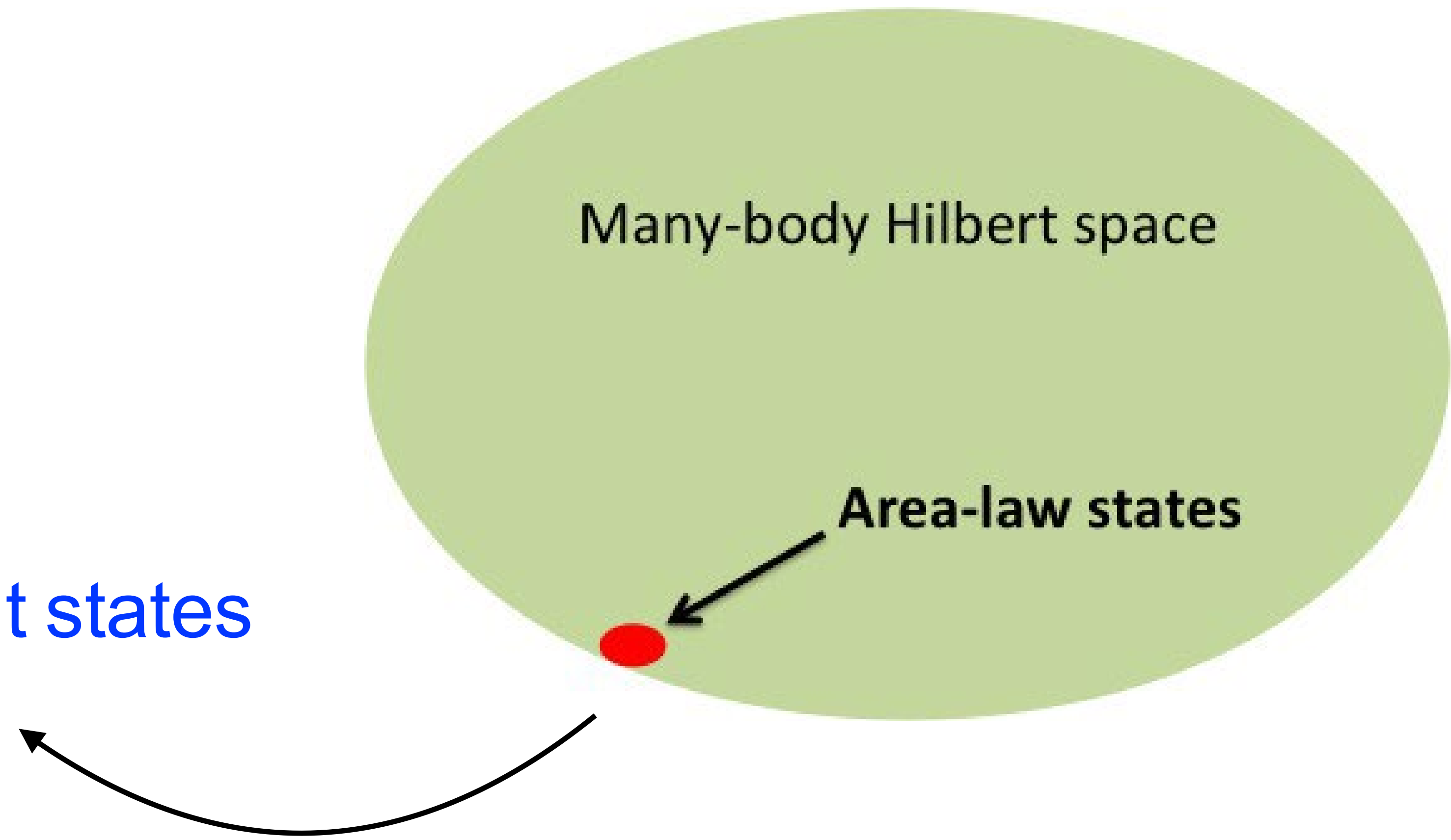
Weakly entangled:
area-law



Highly entangled:
volume-law

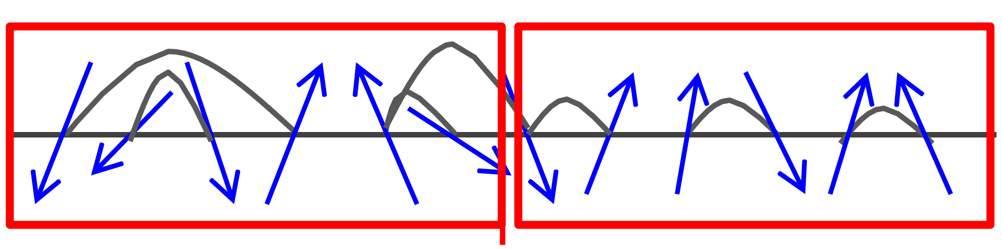


Area- & volume-law states



Also physicaly relevant states

Weakly entangled:
area-law



Highly entangled:
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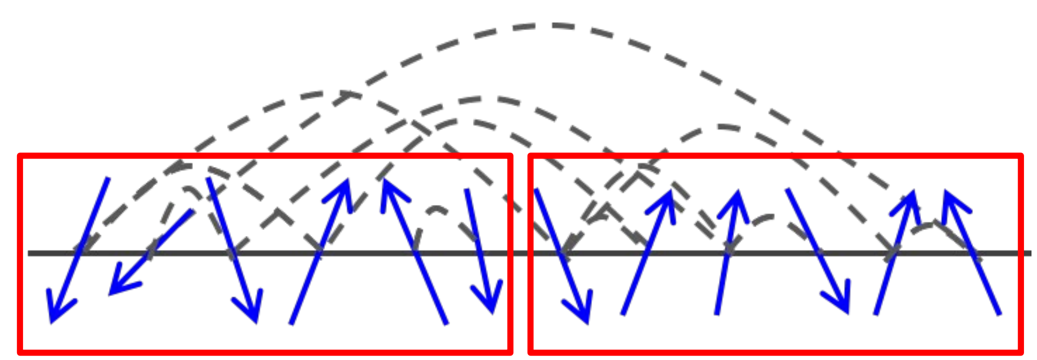
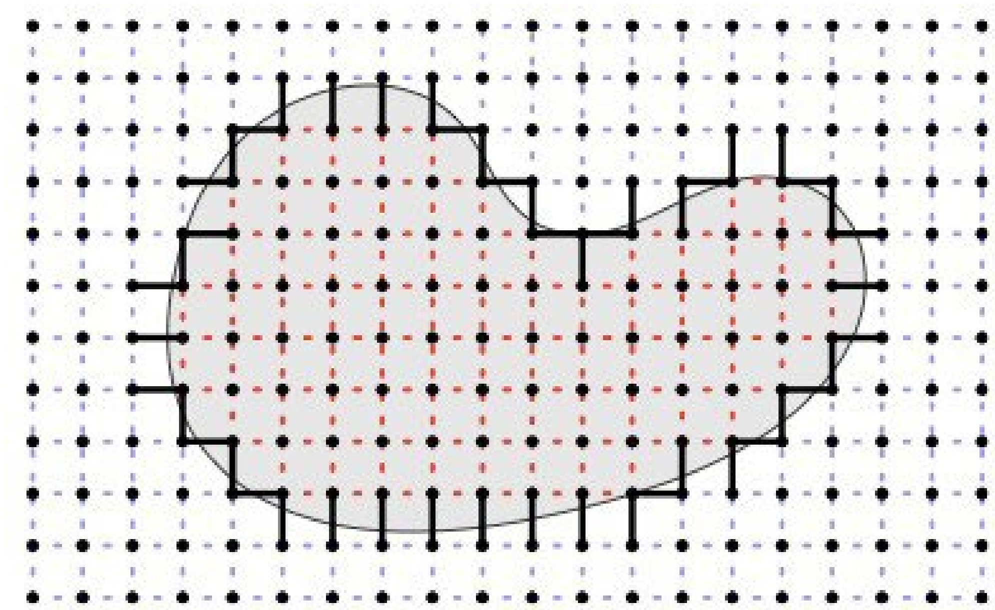


Area-law states

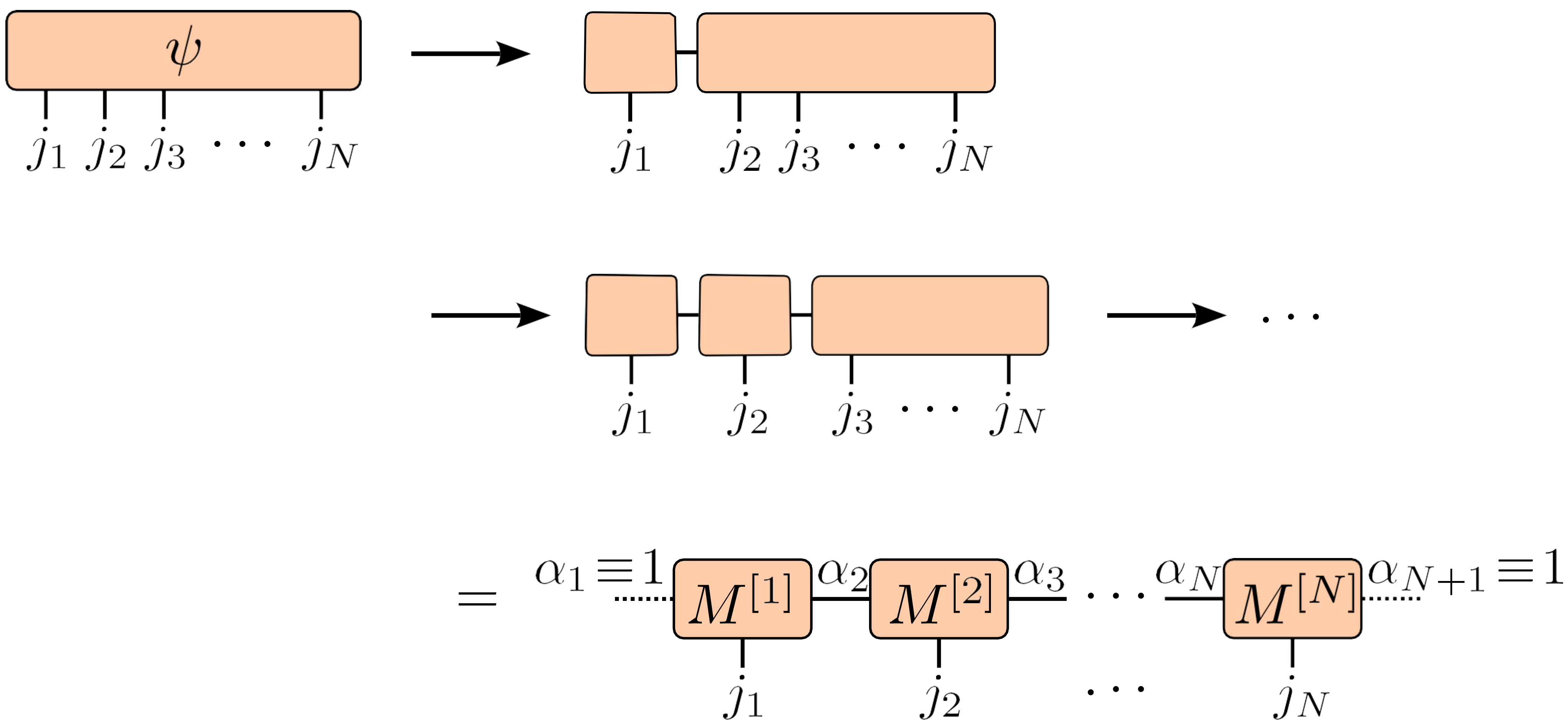
For gapped local Hamiltonians $H = H_1 + \ldots + H_m$, **entanglement entropy** of the ground state scales like **surface area**, rather than volume.



Vidal, Latorre, Rico, Kitaev '02

Inspired by
Holographic Principle and Black hole entropy

Matrix Product state (MPS) representation

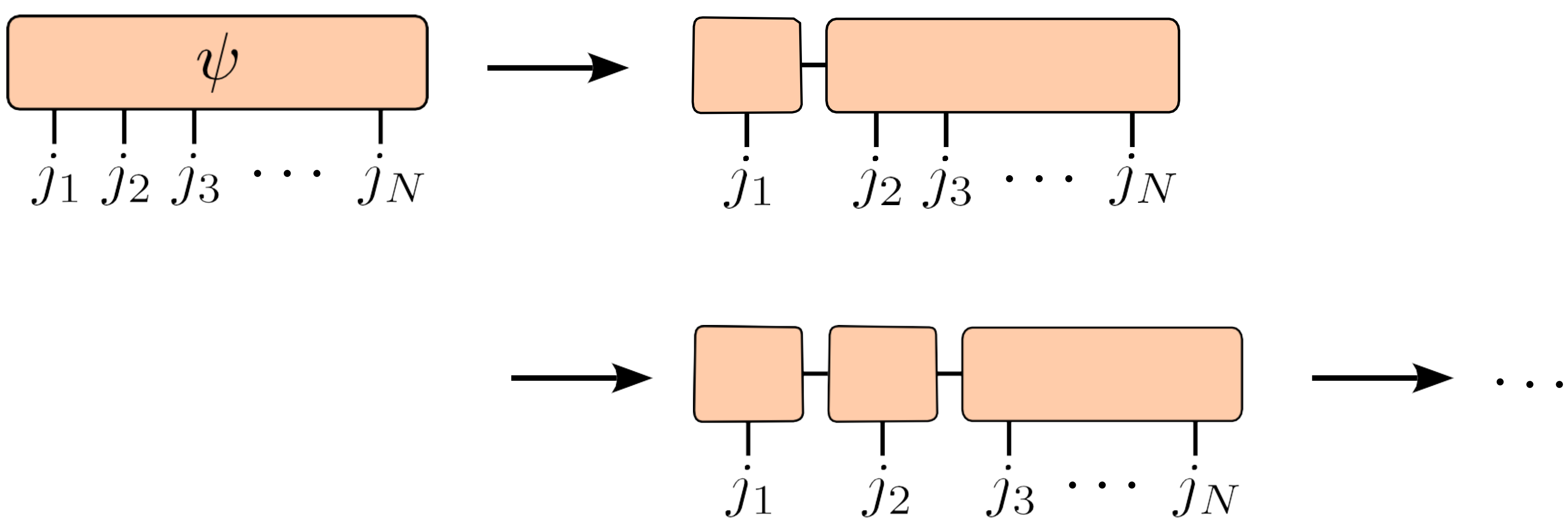


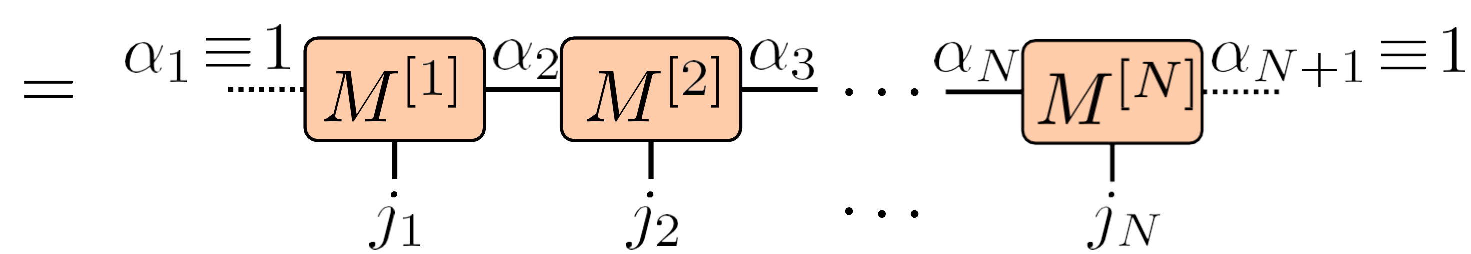
Matrix Product state (MPS) representation

$$| \rangle = \sum_{j_1, j_2 \ldots j_N} {}_{j_1 j_2 \ldots j_n} |j_1, j_2, \ldots, j_N\rangle$$

$$| \rangle = \sum_{j_1, \ldots, j_N} \sum_{\alpha_2, \ldots, \alpha_N} M^{[1]j_1}_{\alpha_1 \alpha_2} M^{[2]j_2}_{\alpha_2 \alpha_3} \ldots M^{[N]j_N}_{\alpha_N \alpha_{N+1}} |j_1, j_2, \ldots, j_N\rangle$$

$$\approx \sum_{j_1, \ldots, j} M^{[1]j_1} M^{[2]j_2} \ldots M^{[N]j_N} |j_1, j_2, \ldots, j_N\rangle .$$





Curious for more?
Don’t miss José Lado's talk!

Condensed matter physics approaches

Compact representation of ground/low energy state
for 1D quantum systems:

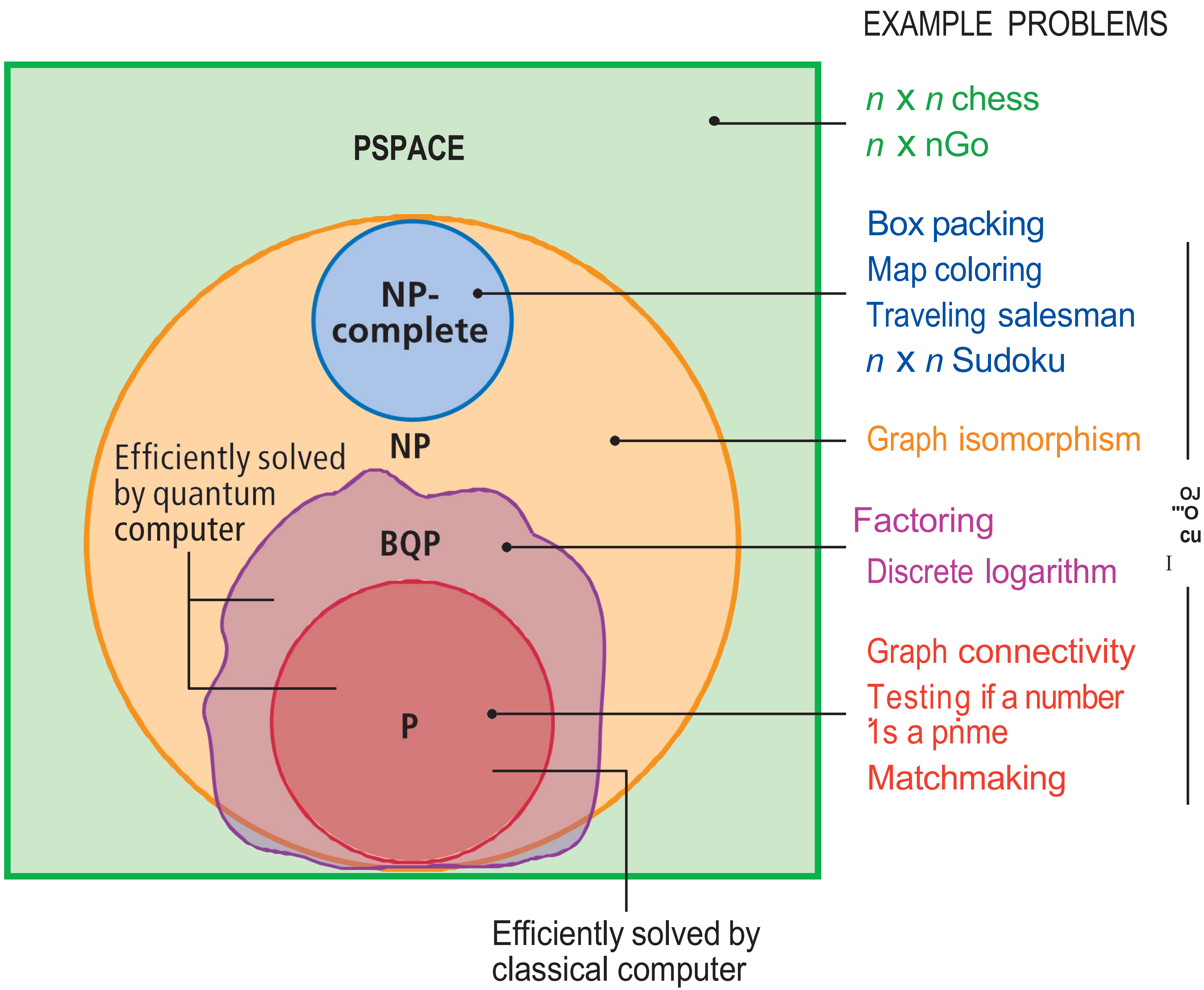
Density Matrix Renormalization Group (DMRG) [White ’92]
and
Matrix Product State (MPS) ansatz

for 2D quantum systems:

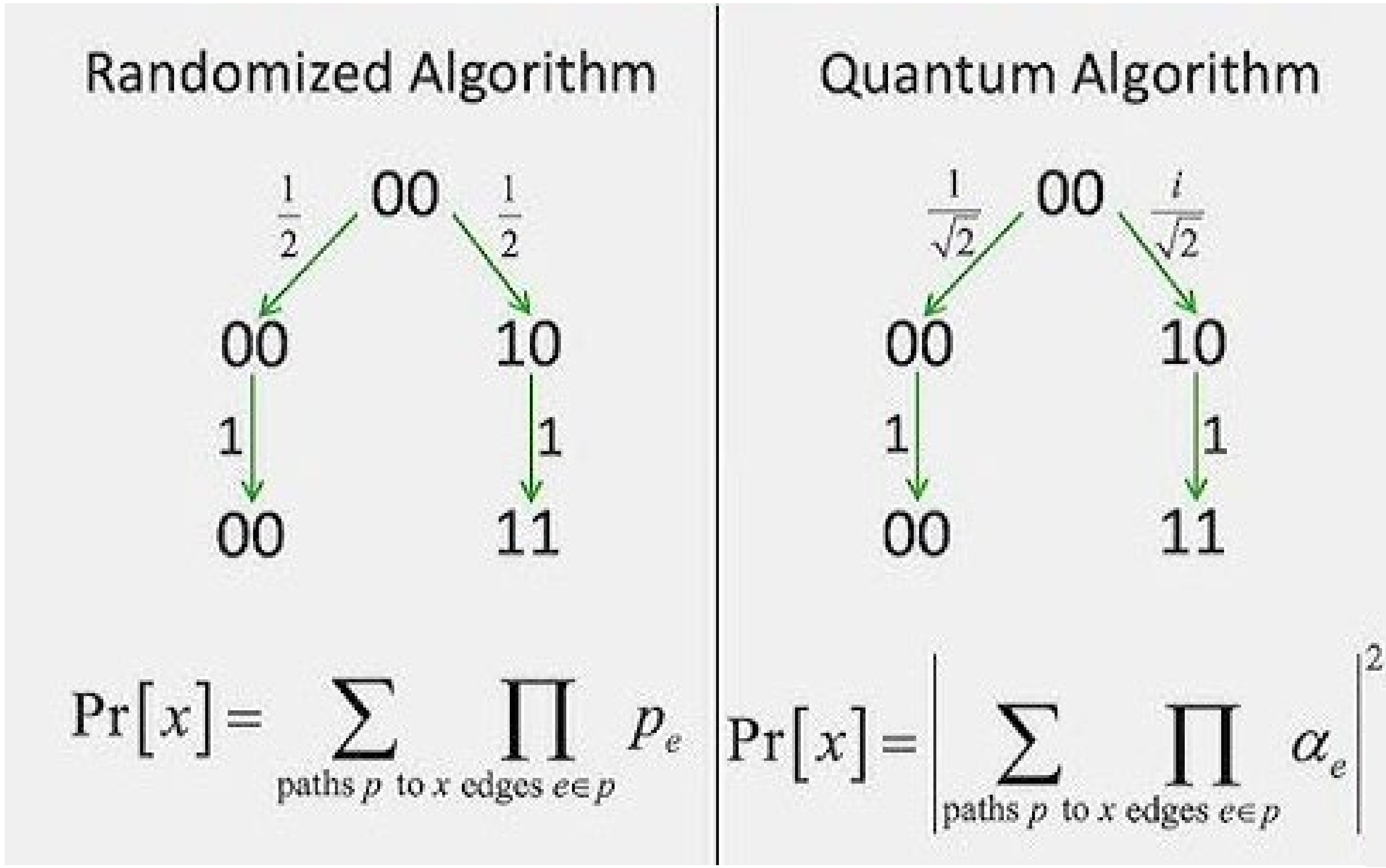
Projected entangled pair states (PEPS) [Vestraete, Cirac ’04]
Multi-scale Entanglement Renormalization Ansatz (MERA) [Vidal ’06]

All under the umbrella of Tensor Networks (TN) [review: Orús ’19]

Computational complexity perpective



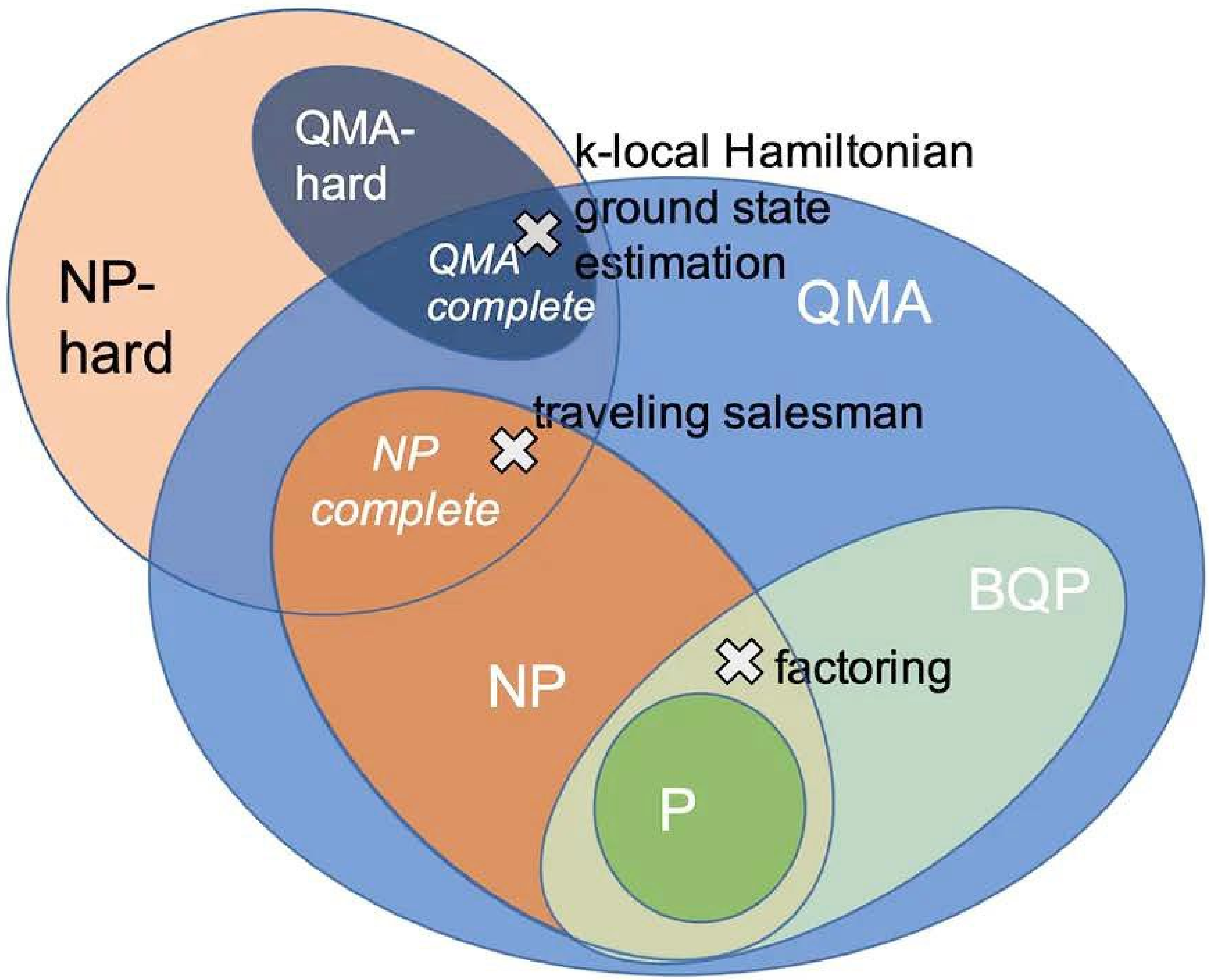
Computational complexity perpective



Quantum physics gives us a bizarre hammer that works well with very special nails (Scott Aaronson)

Computational complexity perpective

Quantum Merlin–Arthur (QMA) complexity classs:
similar to NP but for *quantum* verification



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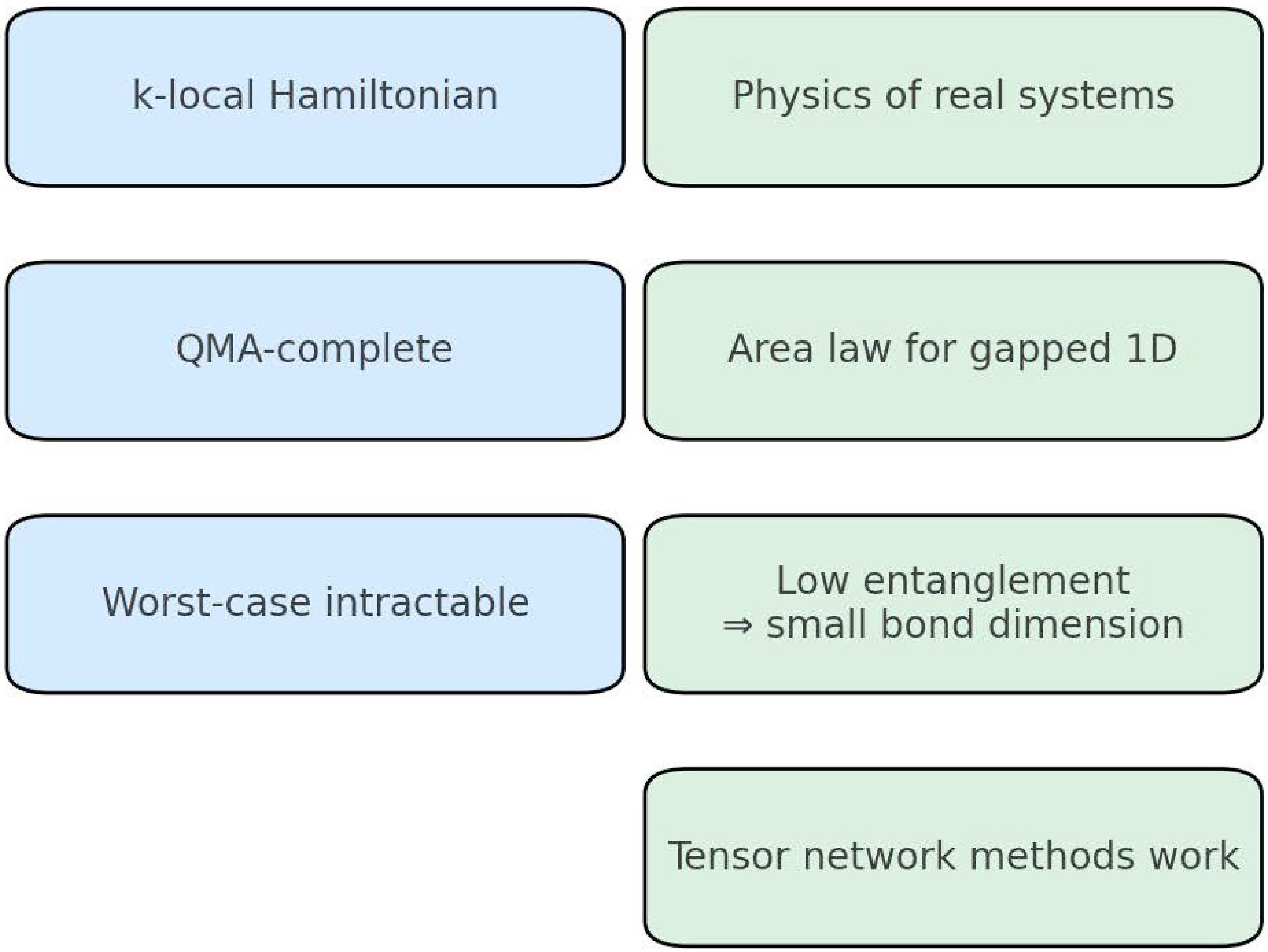


Quantum simulation of manybody systems

Feynman's last dream

RESEARCH ARTICLES

Universal Quantum Simulators

Seth Lloyd

Feynman's 1982 conjecture, that quantum computers can be programmed to simulate any local quantum system, is shown to be correct.

SCIENCE • VOL. 273 • 23 AUGUST 1996

Arguably the most compelling and least overhyped
potential of quantum computing is
to efficiently simulate quantum systems

Quantum simulation of manybody systems

Model System

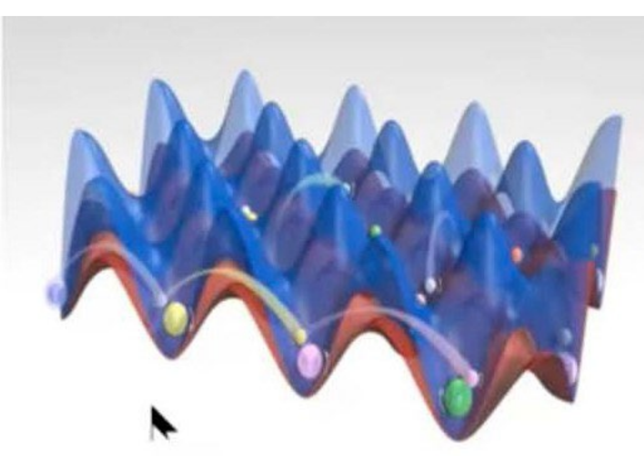
$|\psi'(0)\rangle$ → Evolution → $|\psi'(t)\rangle$

Quantum Simulator

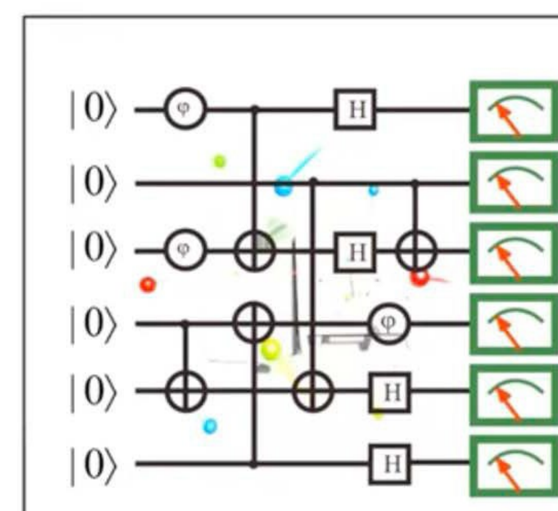
Initialization | $|\psi(0)\rangle$ → Evolution → $|\psi(t)\rangle$ → Measurement

Analog
$U = e^{-iHt}$

Digital
Gate sequence

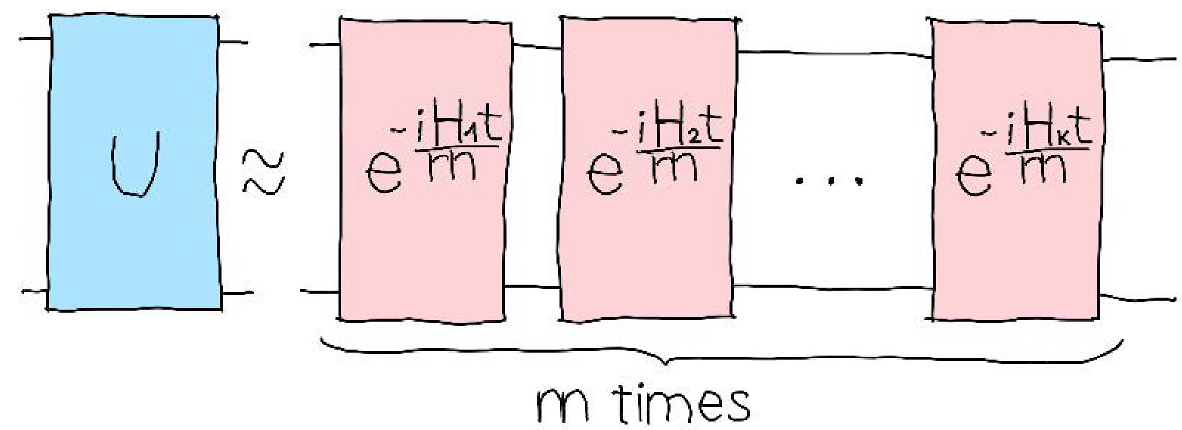


easy-to-prepare
initial state!



Quantum simulation of dynamics: Trotterization

$$H = H_A + H_B$$



$$e^{\rightarrow i(H_A + H_B)t} \rightarrow \left(e^{\rightarrow iH_A \frac{t}{m}} e^{\rightarrow iH_B \frac{t}{m}}\right)^m$$

$$\text{Error} \uparrow \frac{t^2}{2m} \downarrow [H_A, H_B] \downarrow$$

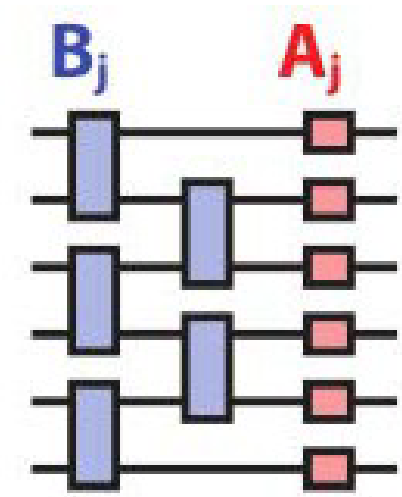
$$m \leftrightarrow \frac{t^2 n}{\omega}$$

$$\text{Gate complexity} \nearrow O\left(n^2 t^2\right)$$

$$e^{\rightarrow iHt} \rightarrow \left(e^{\rightarrow iH_{even} \frac{t}{m}} \ {}^{\rightarrow iH_{odd} \frac{t}{m}}\right)^m$$

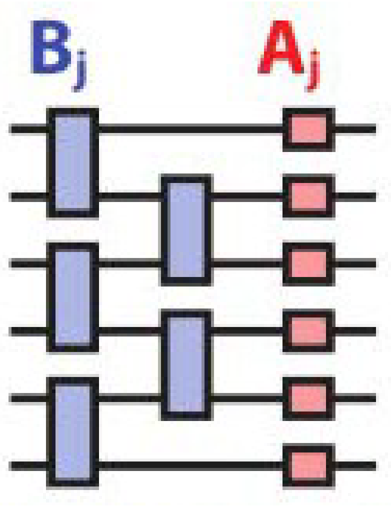
$$A^j = e^{-ih_j \sigma_j^z \Delta t}$$

$$B^j = e^{-i(U\sigma_j^z \sigma_{j+1}^z - J(\sigma_j^x \sigma_{j+1}^x + \sigma_j^y \sigma_{j+1}^y))\Delta t}$$



Mapping Many-Body Hamiltonians to Qubits

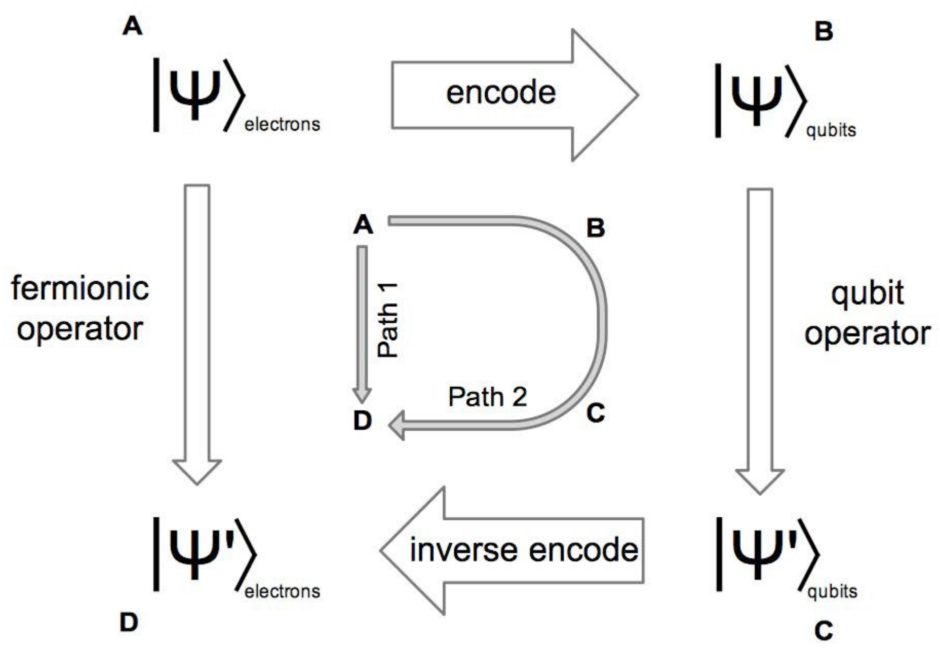
1. Spin Systems (Direct Mapping)



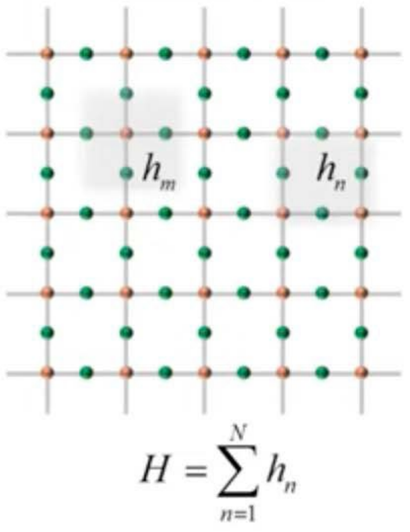
2. Bosonic systems:
   Truncate occupation numbers
   & encode them into qubits

3. Fermionic Systems:

   - Jordan–Wigner Transformation
   - Fermion-to-Majorana Representation
   - Bravyi–Kitaev Transformation:
     logarithmic overhead,
     Efficient for quantum algorithms



Quantum simulation: Ground state



- Prepare a random product state
- Measure the energy

$$| p\rangle = \sum_{n=1}^{2^N} \alpha_n | e_n\rangle$$

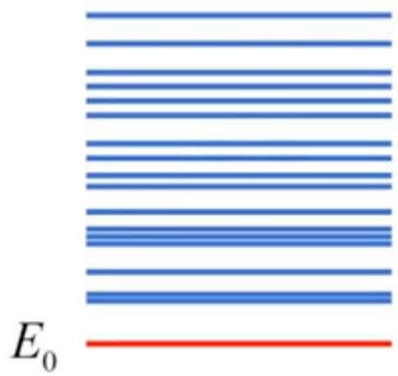
$$H | e_n\rangle = E_n | e_n\rangle$$

success probability

$$P_0 = | \alpha_0 |^2 \approx \frac{1}{2^N}$$

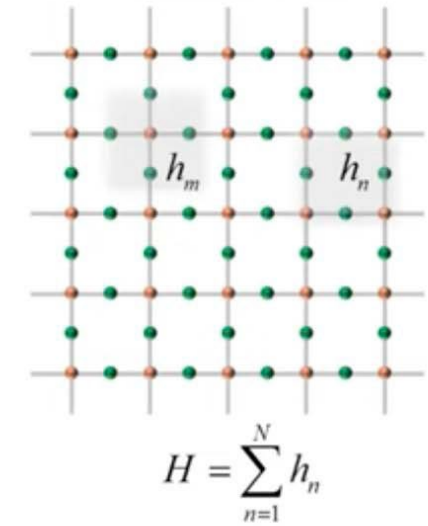
Computational time

$$\tau = \frac{1}{P_0} \approx 2^N$$

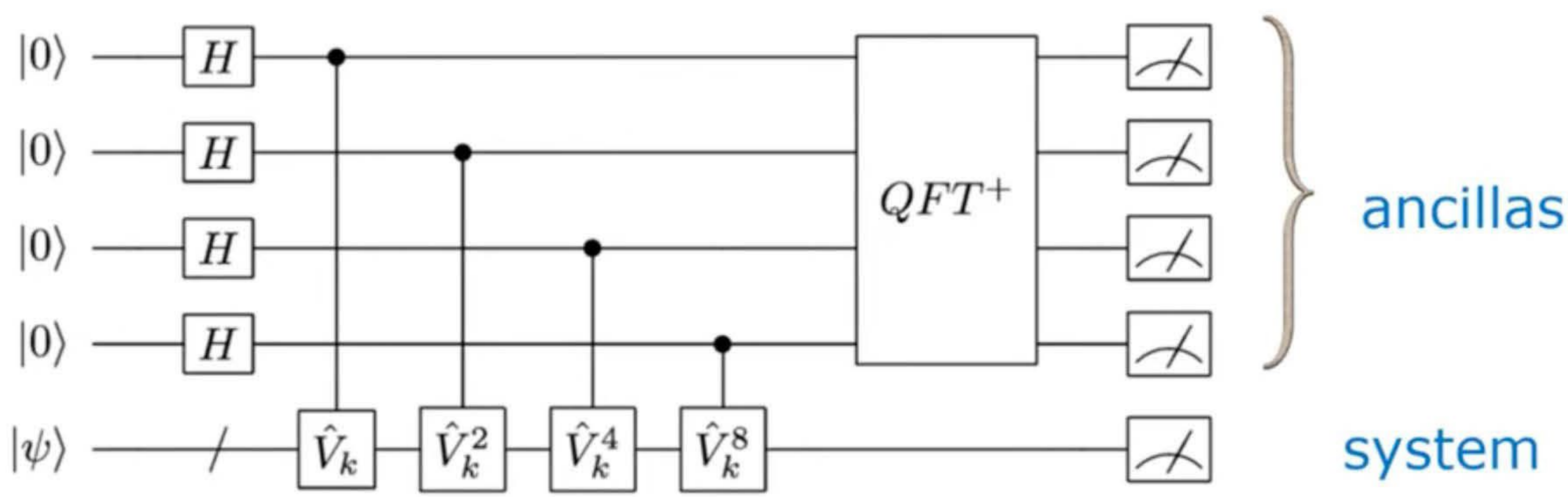


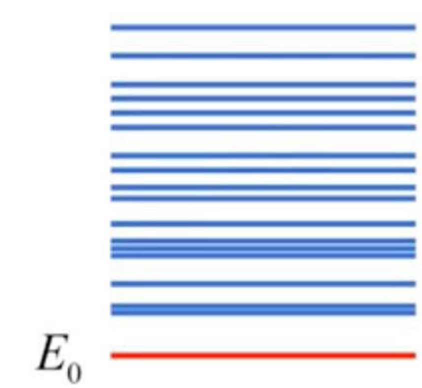
Remeber: It is hard for QC with exponential time scaling,
yet better than superexponential scaling classical algorithms (generic case)

Quantum simulation: Ground state



Quantum phase estimation [Kitaev `95]





$$|e_n\rangle \rightarrow |e_n\rangle |E_n\rangle \qquad H|e_n\rangle = E_n|e_n\rangle$$

e^{-iHt} algorithm for dynamics

Remeber: It is hard for QC with exponential time scaling,
yet better than superexponential scaling classical algorithms (generic case)

Coffee break

Manybody meets QI/QC: key examples

Thermalization in isolated quantum systems:

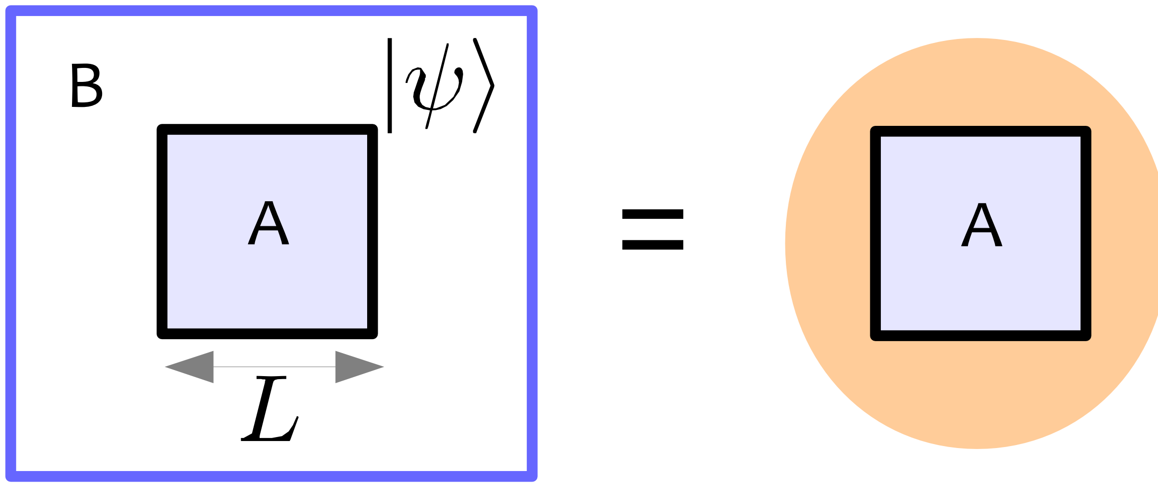
- Quantum chaos,
- Manybody localization (MBL),
- Time crystals,...

Entanglement dynamics in driven systems:

- Floquet quantum systems
- Scrambing in random unitary circuits
- Monitored dynamics & Measurement-induced phases

Eigenstate thermalization hypothesis (ETH)

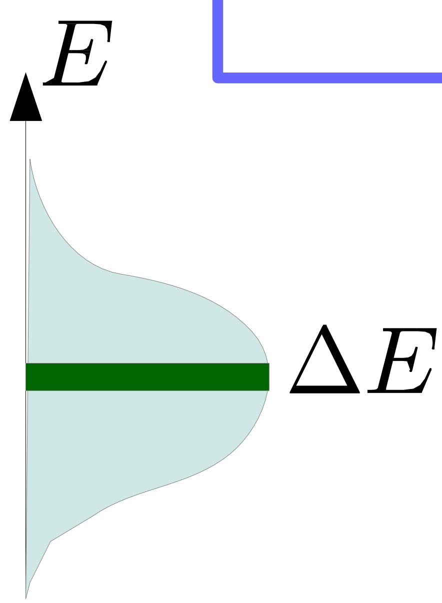
$$H|\psi\rangle = E|\psi\rangle$$



ETH also makes predictions about dynamics:

$$\overline{(O_t - \bar{O})^2} = \mathcal{O}(e^{-S})$$

In?nite time average



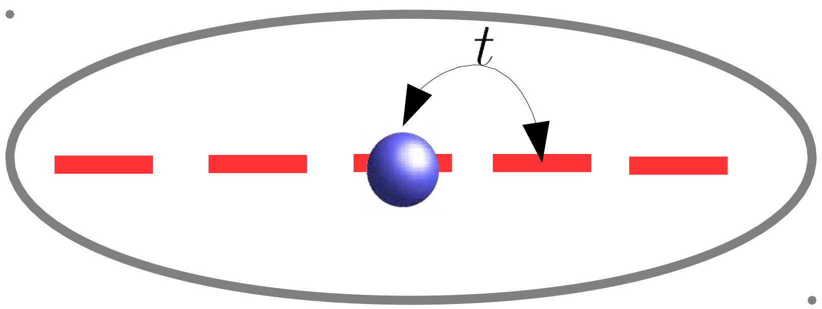
$$\rho_A = \mathrm{tr}_B|\psi\rangle\langle\psi| \approx \rho_{th} = \frac{1}{Z}e^{-\beta H}$$

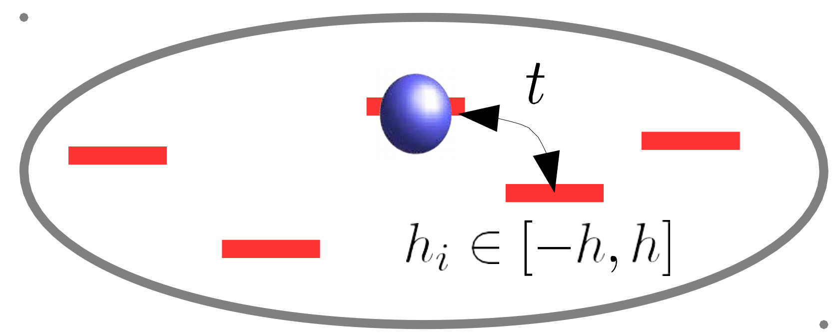
$$\langle\hat{\mathcal{O}}\rangle_E = \langle\hat{\mathcal{O}}\rangle_{th} = \langle\hat{\mathcal{O}}\rangle_{E+\Delta E}$$

In thermalizing system, individual eigenstates look thermal & highly excited eigenstates resemble random vectors

[Deutsch, '91; Srednicki '94]

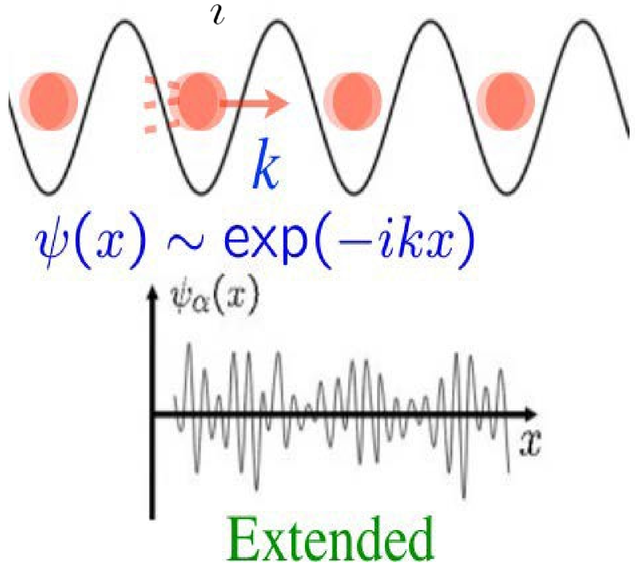
Anderson localization

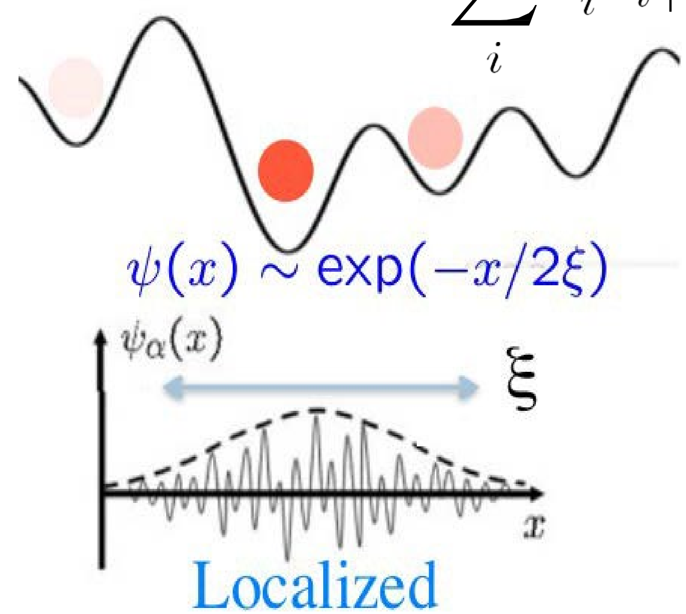


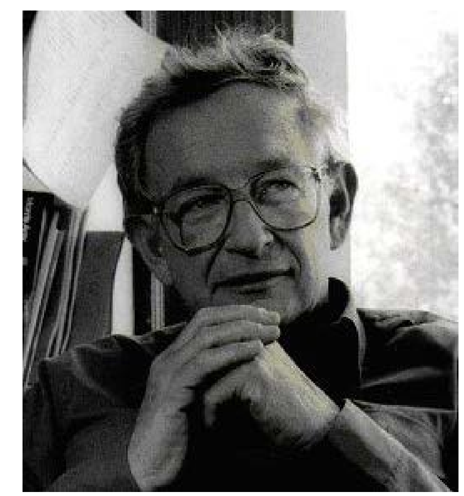


$$H = J \sum_i c_i^\dagger c_{i+1} + h.c.$$

$$H = J \sum_i c_i^\dagger c_{i+1} + h.c. + \sum_i h_i \rho_i$$





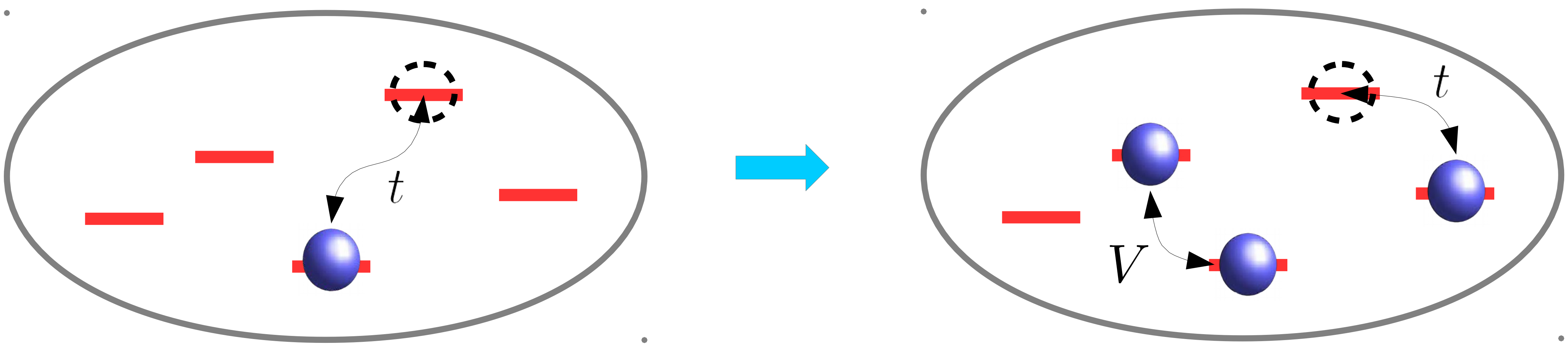


Anderson ('58)

Single-particle states localized with a localization length

States are close in energy, yet far apart In space (no overlap)

Great example of onebody vs. Manybody



$$H = J\sum_i c_i^\dagger c_{i+1} + h.c. + \sum_i h_i \rho_i$$

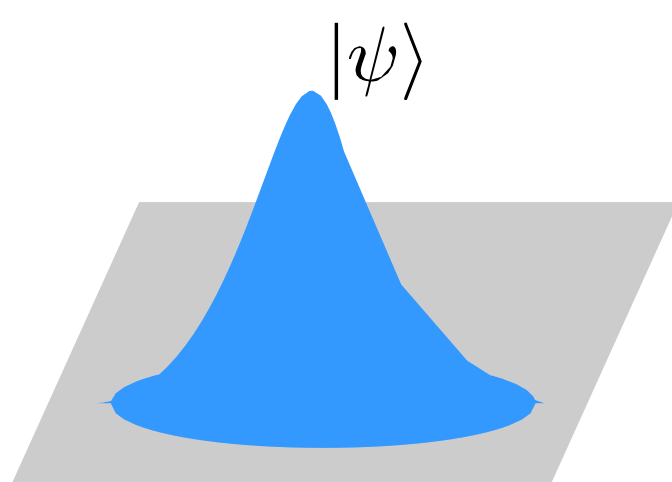
$$H = J\sum_i c_i^\dagger c_{i+1} + h.c. + V\sum_i \rho_i\rho_{i+1} + \sum_i h_i \rho_i$$

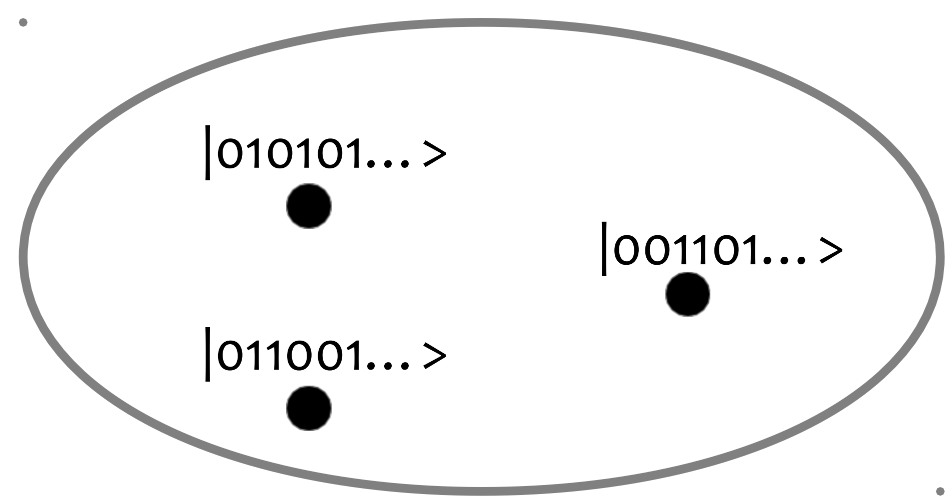
Complexity increases linearly with the number of lattice sites $\sim L$

Complexity increases exponentially with the number of lattice sites $\sim 2^L$

Wavefunction has a direct real space interpretation

Wavefunction lives in Fock space; no direct real space interpretation

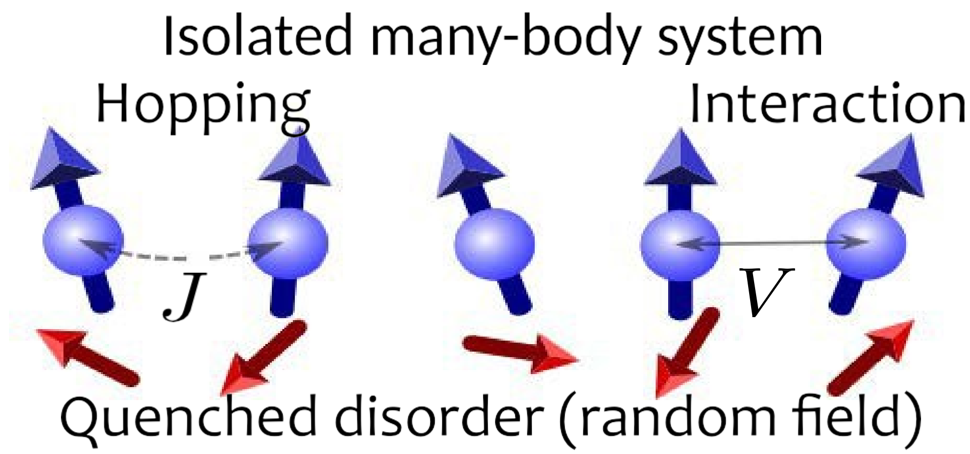




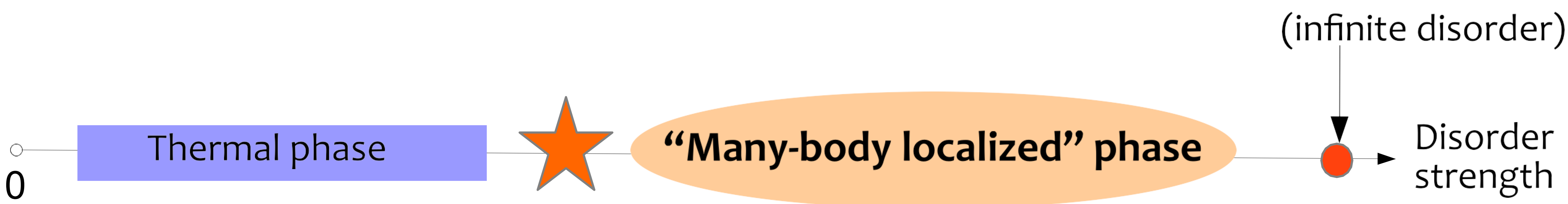
Manybody localization

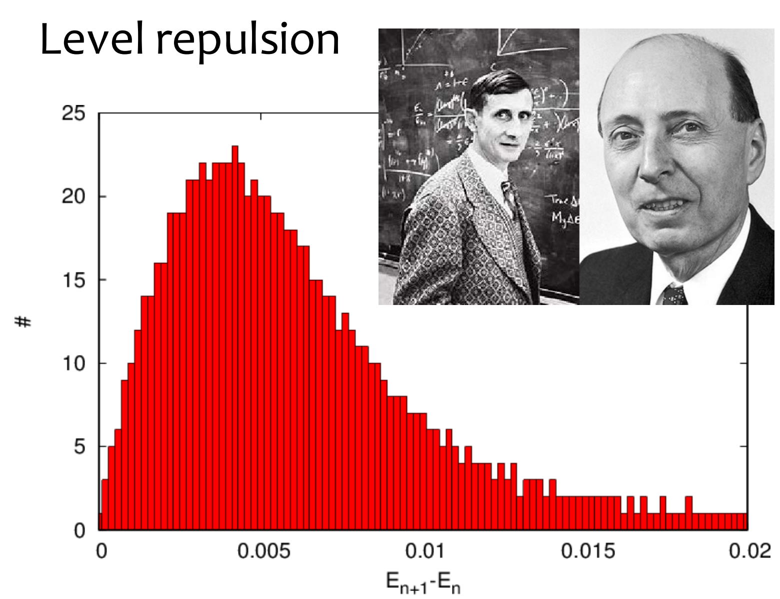
A simple interacting disordered model

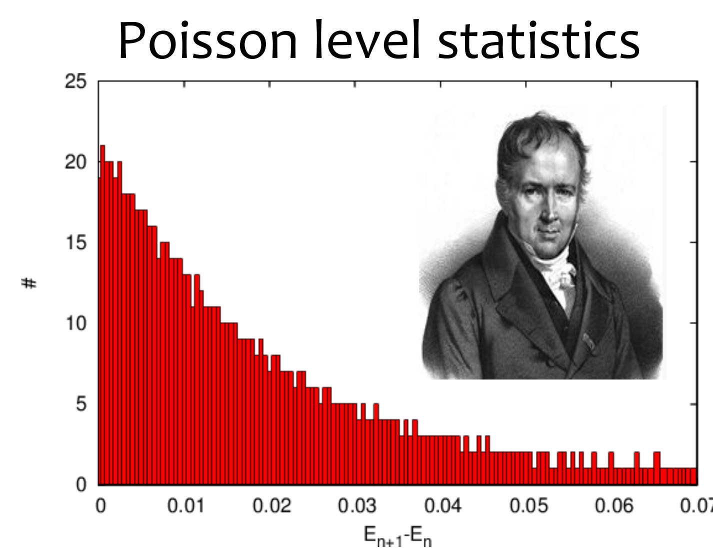
$$H = J\sum_i(\sigma_i^x\sigma_{i+1}^x + \sigma_i^y\sigma_{i+1}^y) + V\sum_i \sigma_i^z\sigma_{i+1}^z + \sum_i h_i\sigma_i^z$$



$$h_i \in [-W, W]$$







Manybody localization

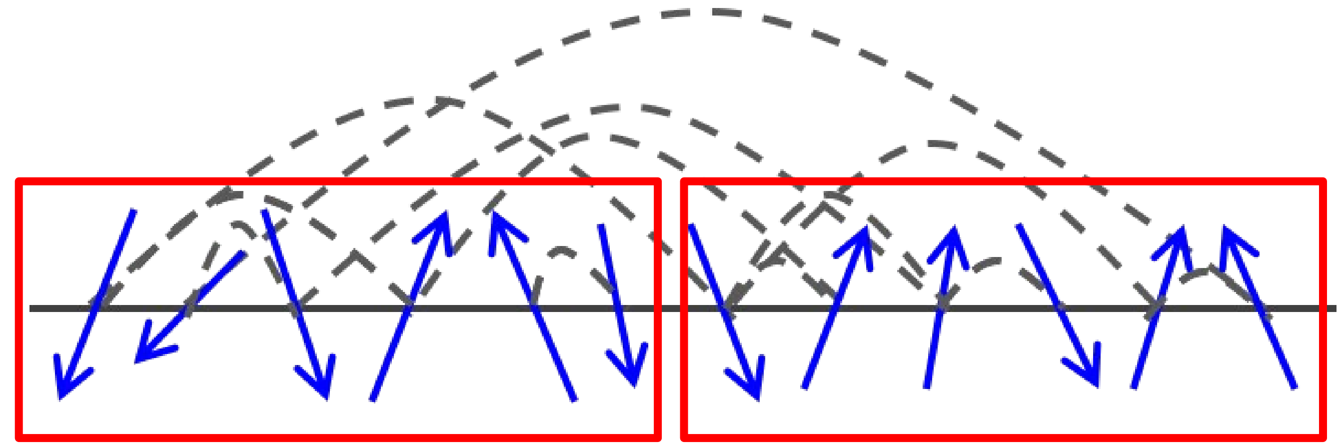
Thermal phase	Anderson	MBL
Memory of initial condition hidden in global operators	Some memory of initial condition persists	Some memory of initial condition persists
ETH true	ETH false	ETH false
Generally non-zero DC conductivity	Zero DC conductivity	Zero DC conductivity
Volume law entanglement in eigenstates	Area law entanglement in eigenstates	Area law entanglement in eigenstates
Power law spreading of entanglement	Finite spreading of entanglement	Logarithmic spreading of entanglement
Local magnetization decays exponentially	Local magnetization does not decay	Local magnetization decays as power law

Landmarks in the history MBL

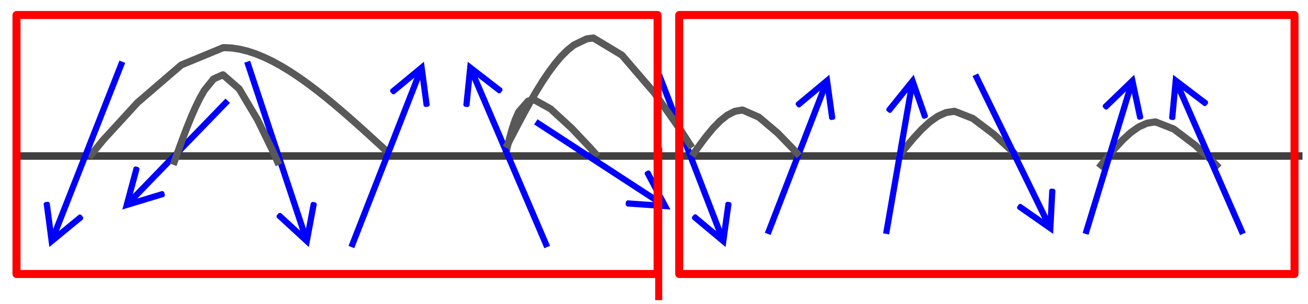
- Anderson introduced the MBL problem but simplified it down to one particle [Anderson, `58]
- Perturbative treatment: localization survives for finite range of interactions [Basko, Aleiner, Altshuler, `06]
- Logarithmic growth of entanglement in MBL systems [Znidaric, Prosen, Prelovsek , `08 ; Bardarson, Pollmann, Moore, `12;]
- Phenomenological picture of local integrals of motion emerges [Serbyn, Papicc, Abanin, `13 ; Huse, Nandkishore, Oganesyan, `14]
- Rigorous proof that LIOMs can be defined in a specific model [Imbrie, `16]
- First experiments looking for MBL in cold atoms and trapped ions [M. Schreiber et al., `15 ; J. Smith et al., `16]

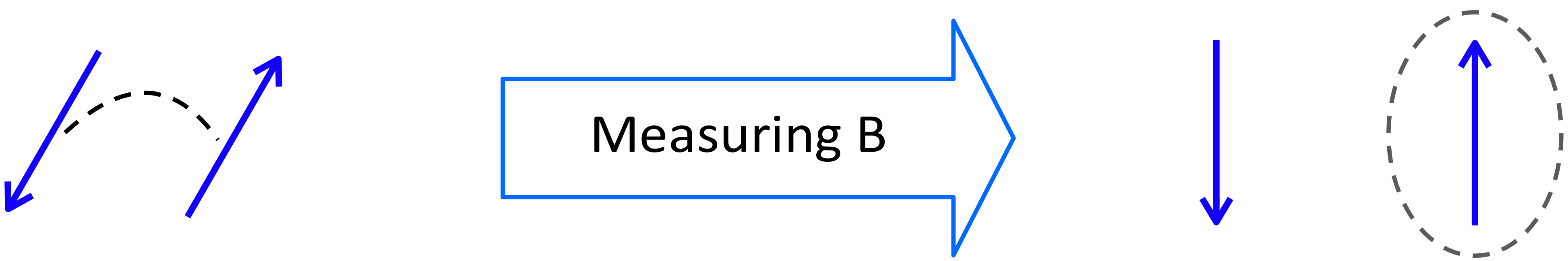
Entanglement vs measurement

Highly entangled:
volume-law



Weakly entangled:
area-law

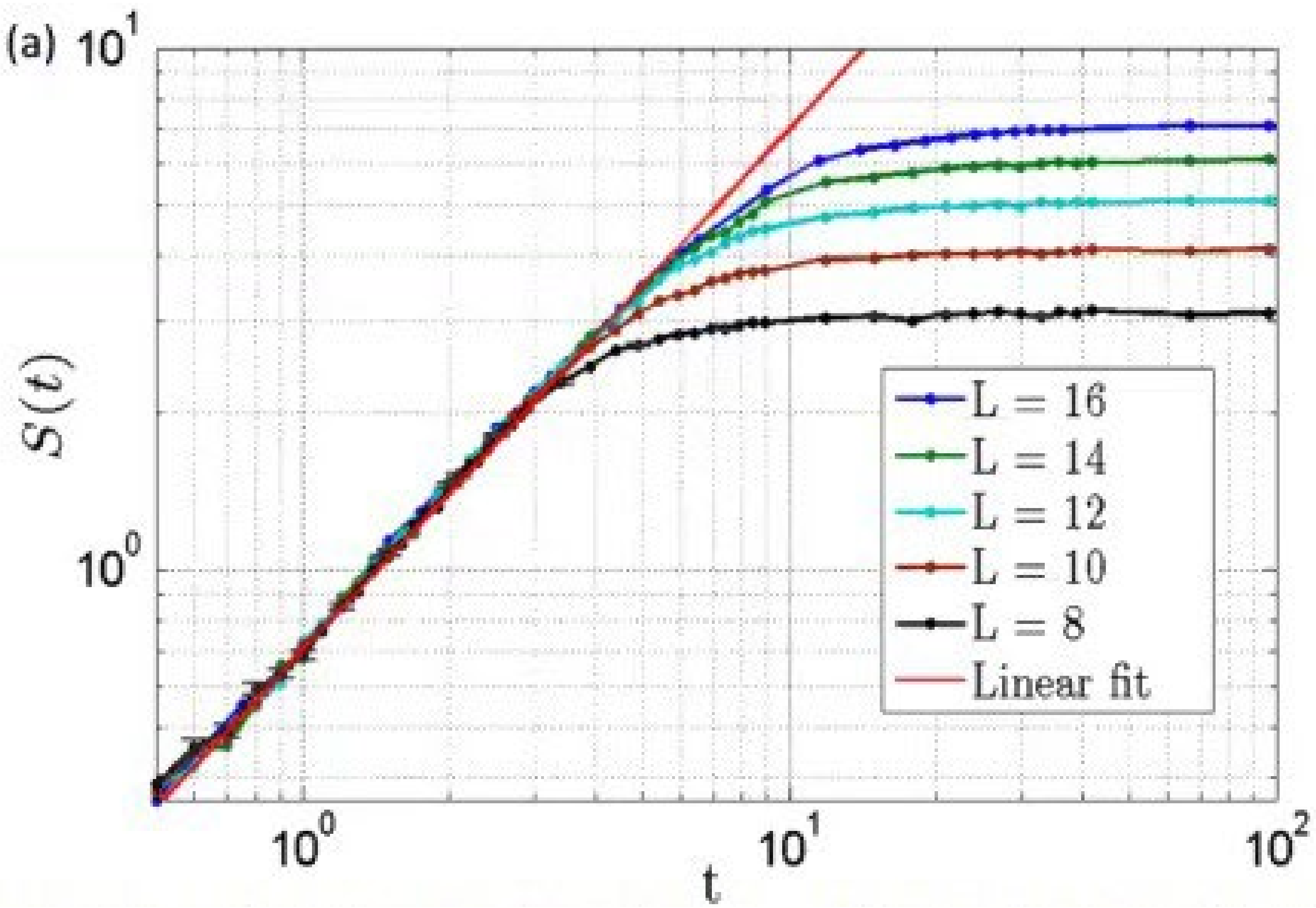


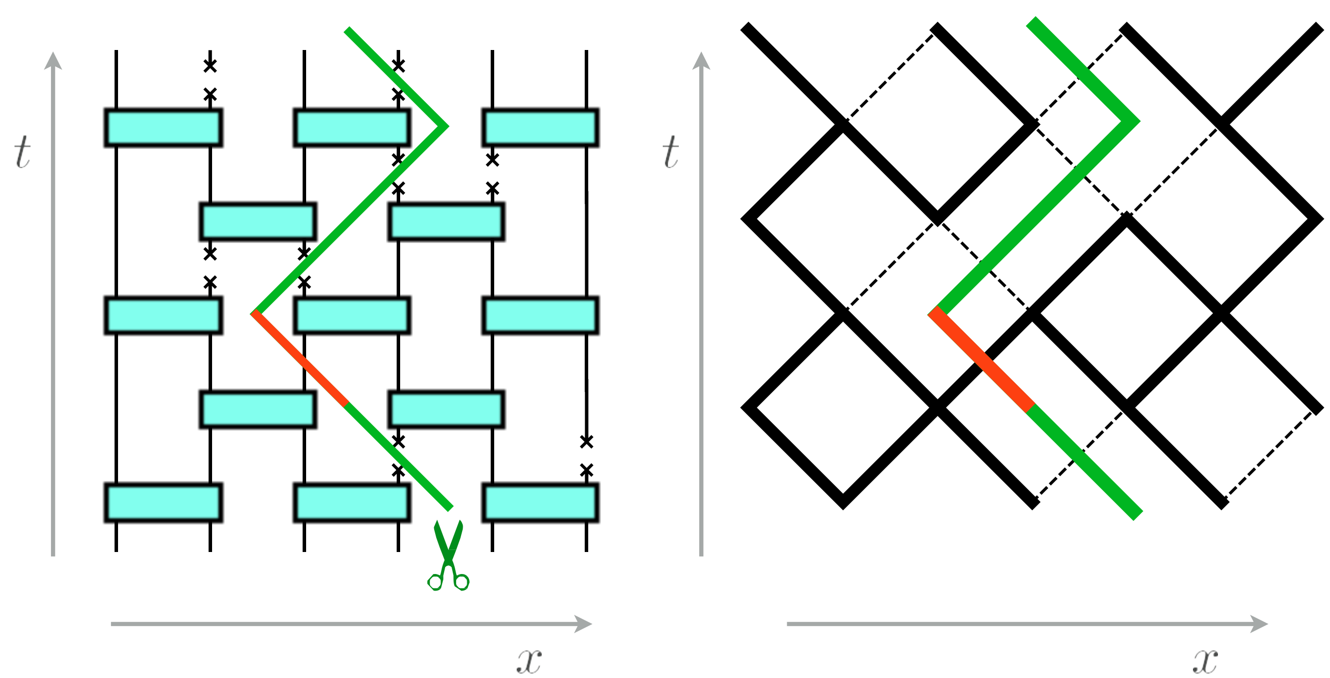


$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow_A \downarrow_B\rangle \quad |\downarrow_A \uparrow_B\rangle\right)$$

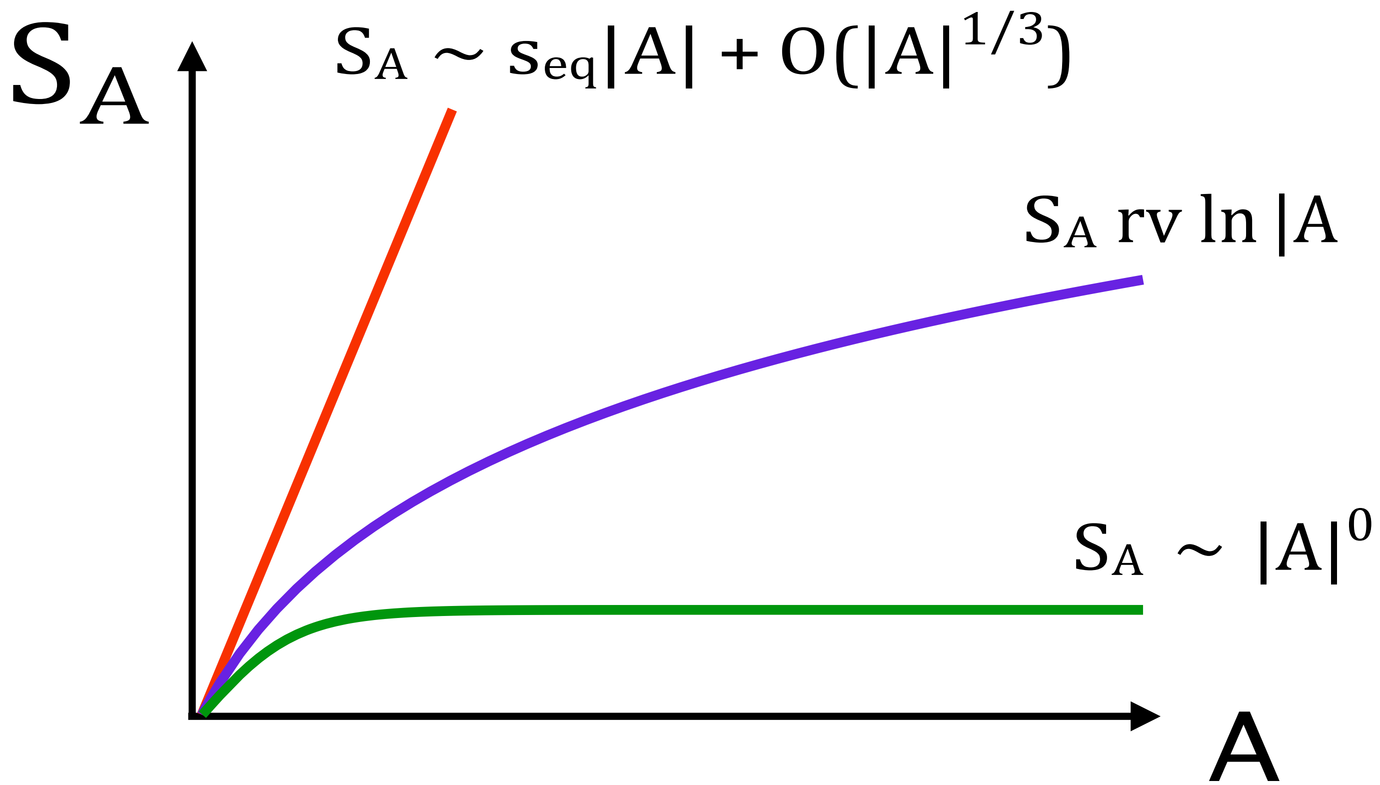
$$|\psi\rangle = |\downarrow_A \uparrow_B\rangle$$

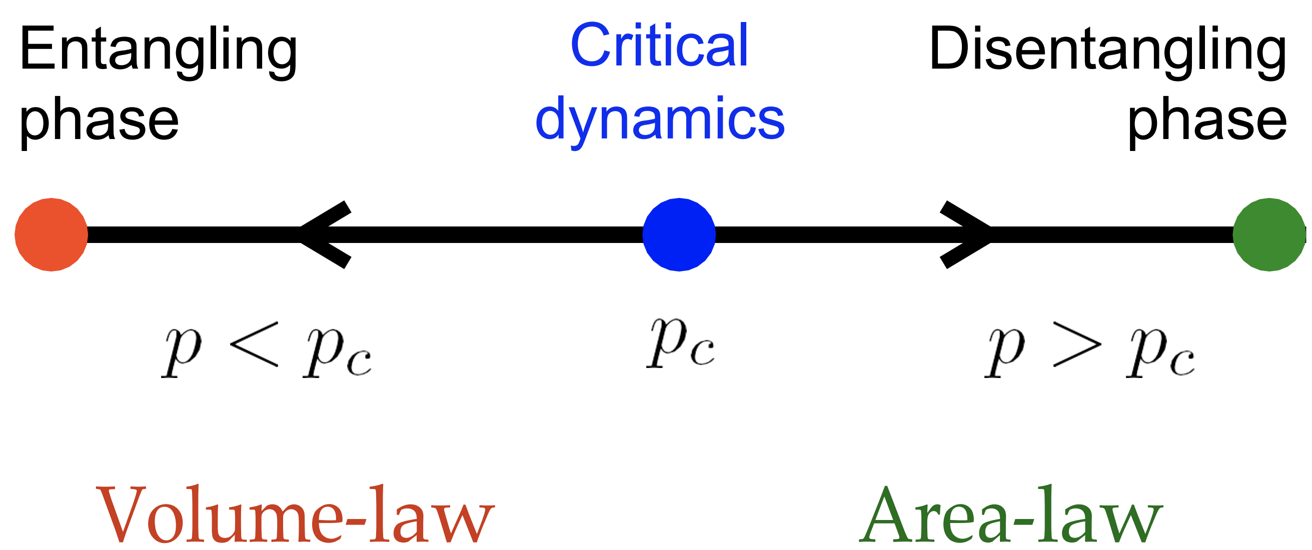
Entanglment growth and interplay with measurement





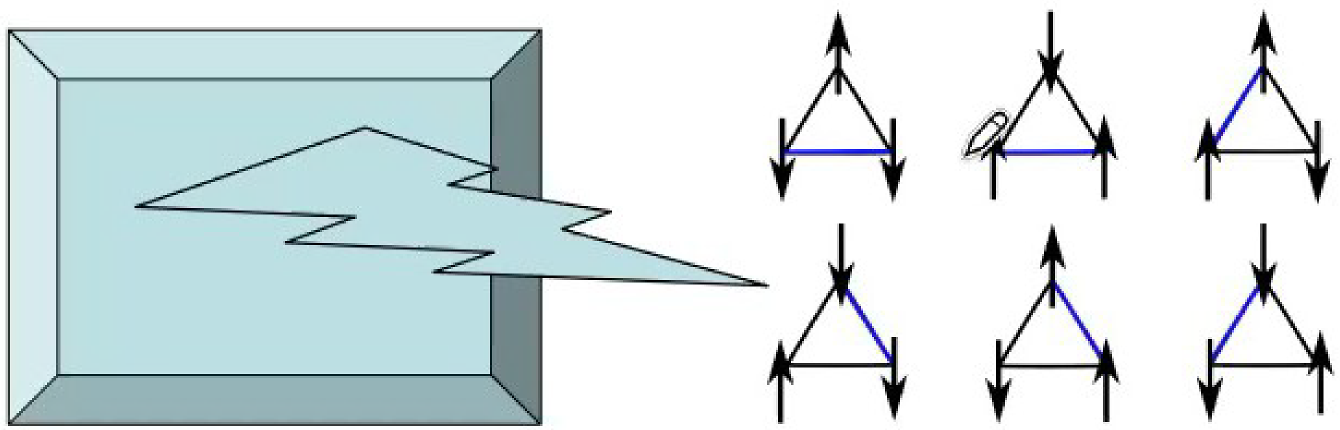
Measurement-induced phase transition (MIPT)





(Traditional) open quantum systems vs. monitored systems

System coupled to a bath (environment)



System is monitored by an "observer"

- Initial pure density matrix becomes mixed
- Environment "measures" system, but results lost
- Decoherence
- Dynamics of density matrix evolves w/ (e.g.) Lindblad equation

Active Decoding with decoherence

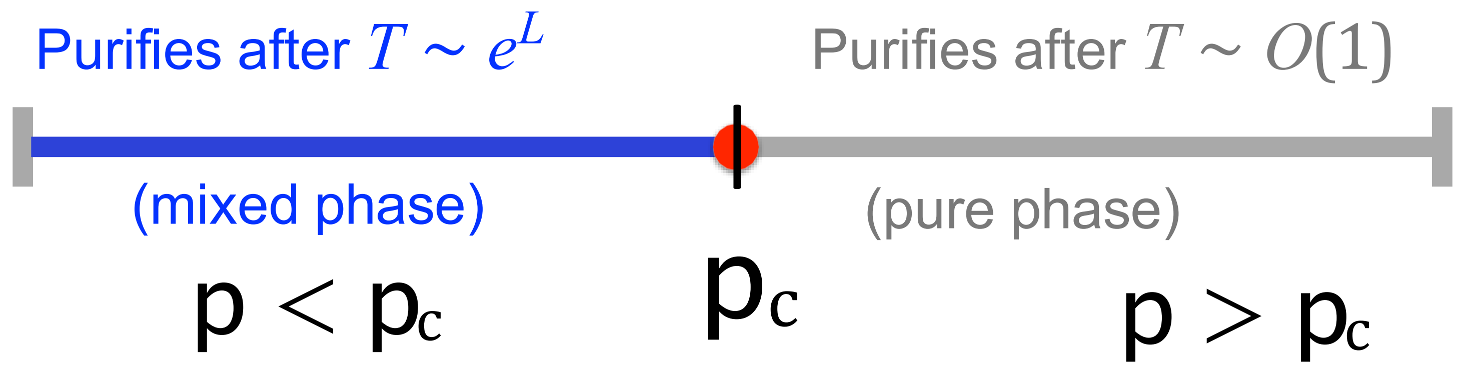
- Initial pure state is measured and stays pure
- "Observer" keeps track of measurements
- Wavefunction evolves as a pure state
- Dynamics described in terms of (wavefunction) quantum trajectories

Measurement-driven entanglement transition

Other manifestaions of MIPT

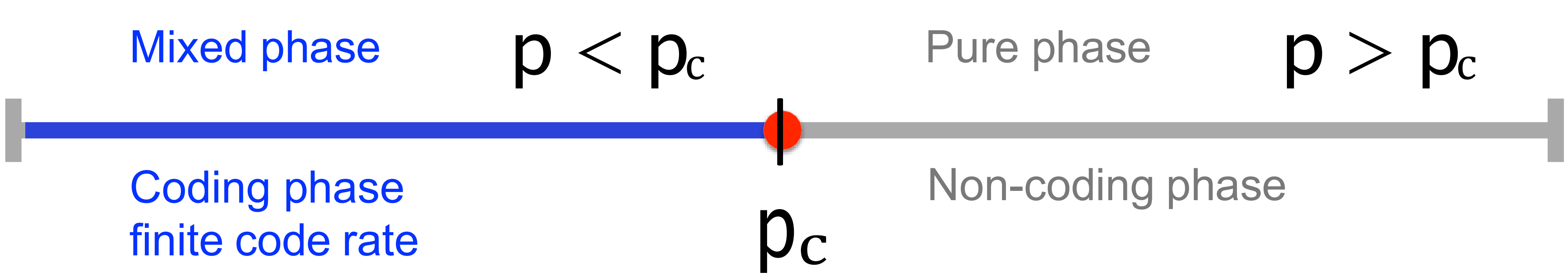
Purification transition

Mixed initial state $\rho_{\text{in}} = \frac{1}{2^L}\mathbb{I}$



Gullans, Huse, PRX (2019)
Noel et. al., Nat. Phys. (2022)

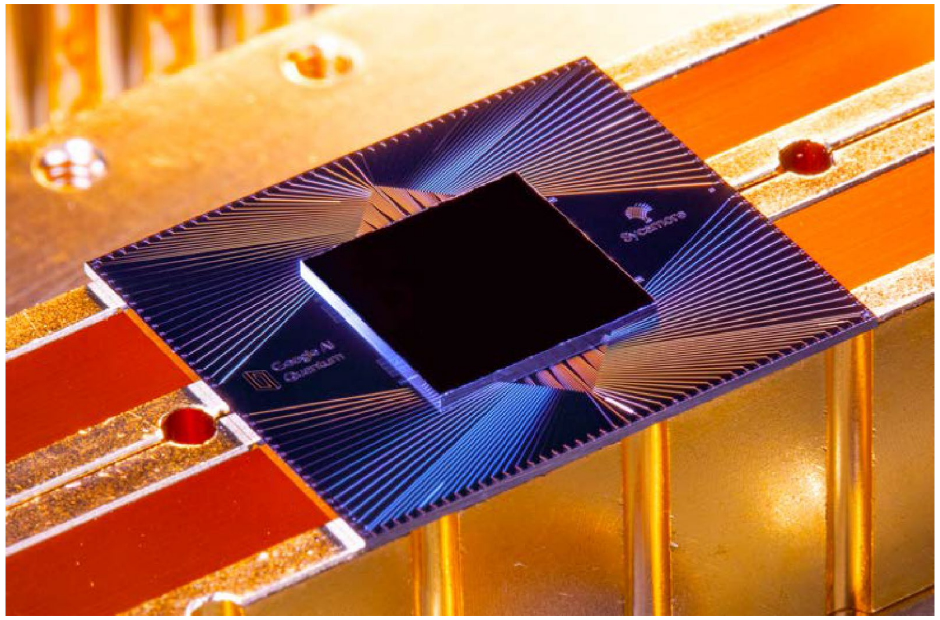
Dynamical quantum memory (dynamical QECC)



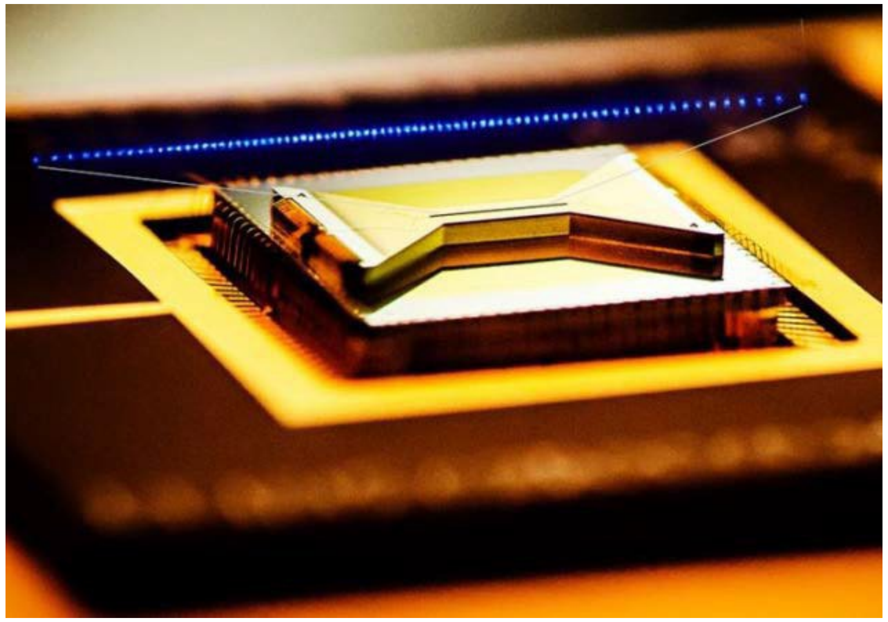
Hayden & Preksill, JHEP (2007)
Choi, Bao, Qi, Altman PRL (2019)

Recent experiments

Superconducting qubits
(IBM-Caltech; Google AI Quantum)



Trapped ions (IonQ)



Noel et al., Nat. Phys. (2022)

Full postselection and tomography

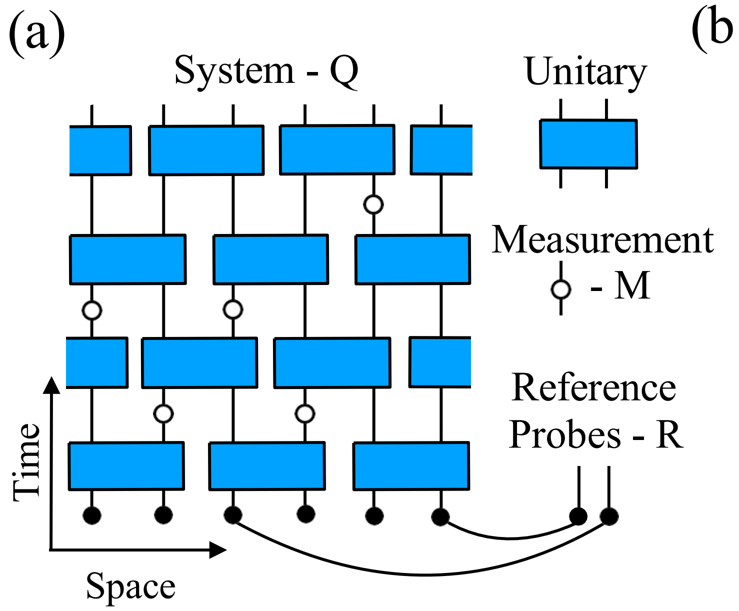
Hoke et al., (Google Quantum AI), Nature (2023);
Koh et al., Nat. Phys. (2023);

Utilizing cross-entropy benchmarking

Kamakari et al., arXiv:2403.00938.

Theory: Li et al., PRL (2023)

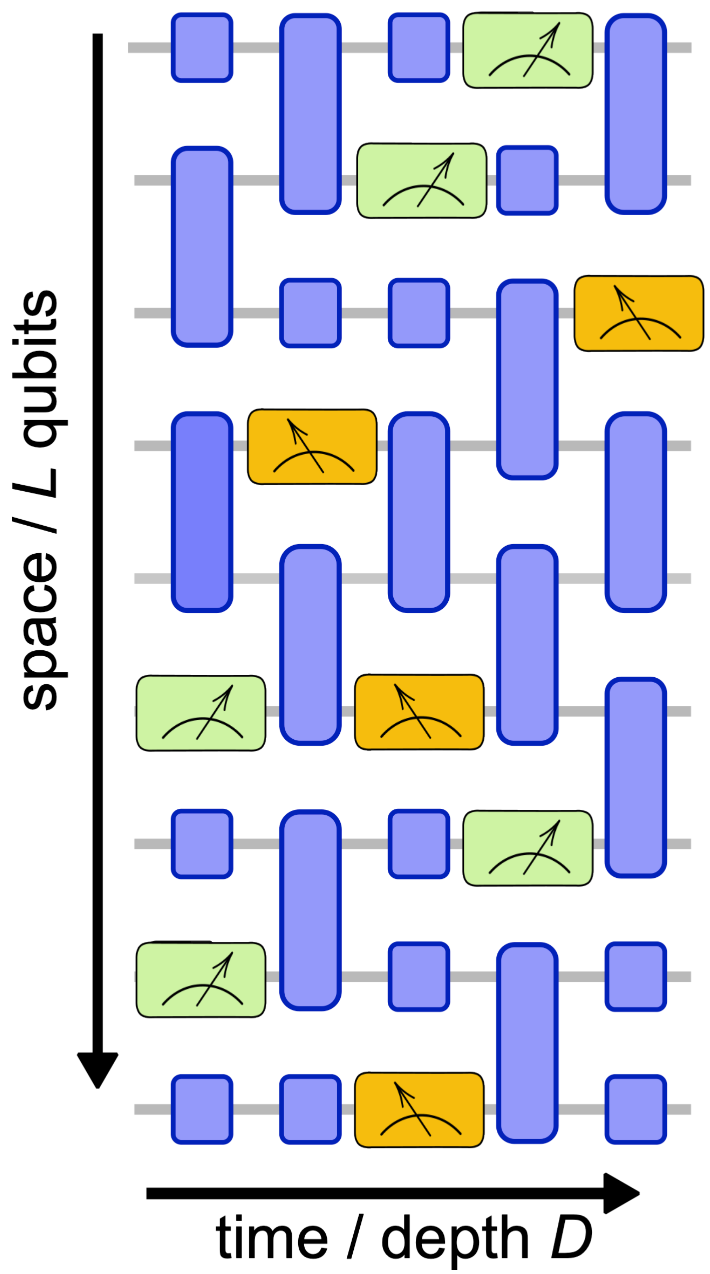
Probing purification transition using single reference qubit



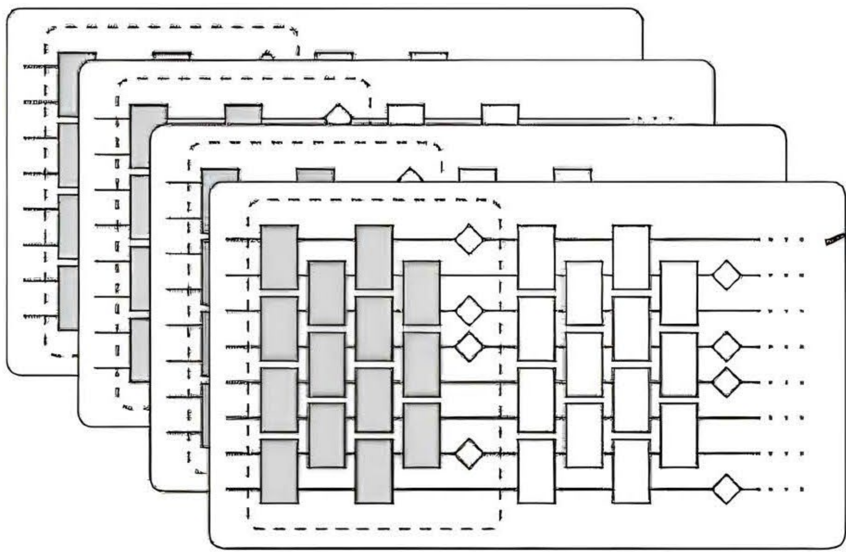
Theory: Gulans & Huse, PRL (2020)

Exponential complexity II: postselection

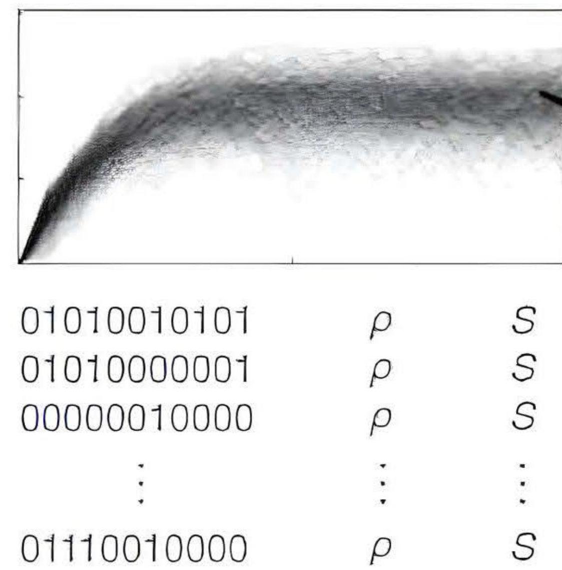
monitored circuit



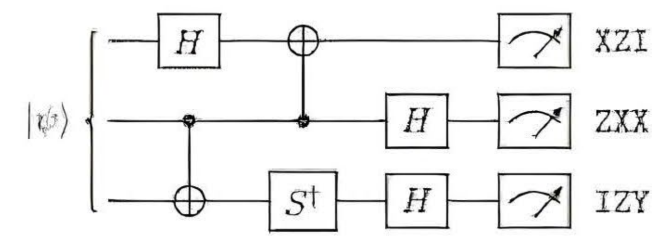
Highly resource-intensive quantum simulations



≥ 50-100 sampled random circuits



~2^{10}-2^{16} trajectories / random circuit



~10 MUBs / trajectory
≥ 1000 shots / MUB

JM Koh et al., Nat. Phys. (2023)

Polynomial:

$O(L^{⌘})$

Exponential:

$O(e^{pLT})$

Exponential:

$O(L^{⌘'} e^{L})$

Summary

- **Manybody systems are the most relevant systems to QI/QC**
- Over the last century the field have become mature
- Effective theories, emergence, fewbody local correlations
- QI/QC provided a deeper take on manybody systems
- Quantum simulation (promises & big questions)
- Quantum advantage in quantum simulation?
- New era of manybody physics?